Reliability Modelling of a Cable Manufacturing Plant With Variation in Demand

Reetu Malhotra, Gulshan Taneja

Dept. of Applied Sciences, Chitkara University, Rajpura, Punjab, India
Dept. of Mathematics, Maharshi Dayanand University, Rohtak, Haryana, India

Abstract

The present paper analyses the reliability and cost-benefit analysis of a cable manufacturing plant for general cable energy with variation in demand. As change in demand affects the production of system also, hence hardly ever, the system needs to be shut down when the number of produces are in excess as compared to those demanded. The expressions for the MTSF, steady state availability; expected busy period for the repairman, expected down time have been obtained using semi-Markov processes and regenerative point technique. Graphical study has also been made which enables us to draw various conclusions of the system.

Keywords

Cable Manufacturing Plant, Regenerative Technique, Semi-Markov Processes, Variation in Demand, Cost-Benefit Analysis

I. Introduction

Since reliability study is considered essential for proper utilization and maintenance of engineering systems and equipments, it has gained much importance among practising engineers and manufacturers. The efficiency of a system is understood to indicate the correctness of the system for the fulfilment of the intended tasks. The suitability of performing specific tasks is primarily determined by the reliability of the system. The literature in reliability is becoming more and more rich day-by-day by various researchers including Murari, K, and Al-Ali, A. A. [1], Gopalan M.N. and Muralidhar M.N. [2], Taneja and Naveen [3], Taneja and Nanda [4], Taneja et al. [5], Parashar and Taneja [6], Goyal et al. [7], Mathew et al. [8], wherein the concepts of instruction, warm standby subject to degradation, three units and repair facilities, patience time with chances of non-availability of expert repairman, one of the two repair policies adopted by the expert repairman after having tried by the ordinary repairman, master slave concept for a PLC system, two unit parallel CC plant system respectively have been taken up. These studies have considered the demand as fixed. However, in most of the practical situations the demand of the units produced is not fixed and hence there is need of varying demand and hence the present paper.

This study investigates the reliability and cost-benefit of a single unit system in which cables are drawn. Copper is a highly ductile material. This means it can easily be drawn into wire. The following measures of the system effectiveness is analysed by making use of semi-Markov processes and regenerative point techniques:

- Mean Time to System Failure
- The Steady State Availability Analysis when Demand ≥ Production
- The Steady State Availability Analysis when Demand < Production
- Busy period of the repairman for repair at t = 0
- Expected number of visits by the repairman at t=0
- Expected down time at t=0
- Profit incurred to the system

II. Nomenclature

Operative state for the system
Failed state for the system
Down state for the system

Op_{dop} Unit is in operative state when demand is not less than production
Op_{dop} Unit is in operative state when demand is less than production
D Unit is in down unit (having no demand)
λ Rate of decrease of demand so as to become less than production
γ_1 Rate of decrease of demand so as to become less than production
γ_2 Rate of increase of demand so as to become greater than production
γ_3 Rate of going from upset state to downstate (reason behind this is that the demand is less than production & production goes on increasing and as a result we have lot of produces in the stock. This production needs to be stopped.
γ_4 Rate of change of state from down to up when there is no produce with the system and demand is there.
Fr Failed unit under repair
p_1 Probability that during the repair time demand is greater than or equal to production
p_2 Probability that during the repair time demand is less than production
A_0^d Steady state availability of the system when demand is greater than or equal to production
A_0^p Steady state availability of the system when demand is less than production
B_0^p(t) Busy period of the repairman for repair at t = 0
V_0^p(t) Expected number of visits by the repairman at t=0
DT_0^p(t) Expected down time at t=0
M_i(t) Probability that system up initially in regenerative state i and then at time t without passing through any other regenerative state
m_i Contribution to mean sojourn time in regenerative state i before transiting to regenerative state j without visiting to any other state
μ_i(t) Mean sojourn time in regenerative state before transiting to any other state
* Symbol for Laplace transforms
** Symbol for Laplace Stieltjes transforms
© Symbol for Laplace convolution
s Symbol for Stieltjes convolution
qij(t) p.d.f. of first passage time from a regenerative state i to a regenerative state j or to a regenerative state j to a regenerative state k.
III. Model Description and Assumptions
1. Completion of the repair process puts the unit back into operation.
2. As soon as the unit fails, it is undertaken for repair.
3. All the random variables are independent.
4. The breakdown times are assumed to be exponentially distributed whereas the repair time distributions are arbitrary.

IV. Transition Probability and Mean Sojourn Times
The epochs of entry into states $S_0$, $S_1$, and $S_2$ are regenerative points and thus are regenerative states. States $S_0$, $S_2$, and $S_4$ are failed states. The non-zero elements $p_{ij}$ are:

- $p_{01} = \gamma_1 / (\lambda + \gamma_1)$
- $p_{02} = \gamma / (\lambda + \gamma_1)$
- $p_{03} = \gamma_2 / (\lambda + \gamma_1 + \gamma_3)$
- $p_{04} = \lambda / (\lambda + \gamma_1 + \gamma_3)$
- $p_{20} = 1$
- $p_{21} = 1$
- $p_{40} = 1$
- $p_{41} = 1$

The mean sojourn time $m_i$ in state $i$, where $i$ is already specified, is $1 / (\lambda + \gamma_i)$.

V. Measures of System Effectiveness
A. Mean Time to System Failure
To determine the mean time to system failure (MTSF) of the system, when the system starts from the state $'0'$ is:

$$\text{MTSF} = \lim_{s \to 0} \left( 1 - f_0^(**)(s) \right) / s$$

Using L'Hospital rule and putting the value of $f_0^{**}(s)$ from eqn we have

$$\text{MTSF} = N / D$$

where $N = p_0 + p_{01} \mu_1 + p_{03} \mu_3$ and $D = p_{01} \mu_1 + p_{02}$

B. Availability Analysis When Demand is Less Than Production
In steady-state, the availability of the system is given by:

$$A_0 = \lim_{s \to 0} (SA_0^*(s)) = N_2 / D_1$$

where $N_1 = p_0 \mu_1$ and $D_1 = (1- p_{04} \mu_4) \mu_0 + p_{01} \mu_1 + p_{02} (1- p_{04} \mu_4) \mu_2 + (p_{01} \mu_3) \mu_3 + (p_{04} \mu_4) \mu_4$

C. Availability Analysis When Demand is Greater Than or Equal to Production
The availability of the system, in steady-state, is given by

$$A_0 = \lim_{s \to 0} (SA_0^*(s)) = N_2 / D_1$$

where $N_2 = (1-p_{04} \mu_4) \mu_0 + p_{01} \mu_1 + p_{02} (1-p_{04} \mu_4) \mu_2 + (p_{01} \mu_3) \mu_3 + (p_{04} \mu_4) \mu_4$

D. Busy Period Analysis of the Repairman
In steady-state, the total fraction of the time for which the system is in repair of the repairman is given by

$$B_0 = \lim_{s \to 0} (sV_0^{**}(s)) = N_4 / D_4$$

where $N_4 = p_{02} (1- p_{04} \mu_4) + p_{01} \mu_4$ and $D_4$ is already specified.

E. Number of Visits by the Repairman
In steady-state, the number of visit per unit time by the ordinary repairman is given by

$$V_0 = \lim_{s \to 0} (sV_0^{**}(s)) = N_4 / D_4$$

where $N_4 = p_{02} (1- p_{04} \mu_4) + p_{01} \mu_4$, and $D_4$ is already specified.

F. Expected Down Time
In steady-state, the total fraction of the time for which the system is in down state given by

$$DT_0 = \lim_{s \to 0} (sDT_0^{**}(s)) = N_4 / D_4$$

where $N_4 = p_{01} \mu_4$ and $D_4$ is already specified.

VI. Transition Diagram
Now in the given transition diagram, $S_0$, $S_2$, and $S_4$ are regenerative states indicating $Op$, $Op$, and $D$, whereas States $S_2$ and $S_4$ are failed states indicating $F_r$.

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**Fig. 1: State Transition Diagram**
VII. Cost-Benefit Analysis

Profit (P) =
\[ C_0 A_0 t - C_1 A_0 p - C_2 B_0 - C_3 V_0 - C_4 D T_0 \]
where
- \( C_0 \) = Revenue per unit up time when demand is greater than or equal to production
- \( C_1 \) = Revenue per unit up time when demand is less than or equal to production
- \( C_2 \) = Cost per unit up time for engaging the repairman for repair
- \( C_3 \) = Cost per visit of the repairman
- \( C_4 \) = Loss per unit time during the system remains down

VIII. Graphical Analysis

For the particular case, the rate of repairable failure is assumed to be exponentially distributed i.e.
Let us take \( g(t) = \alpha e^{-\alpha t} \)
g*'(0) = \(-1/\alpha\) and \( p_1 = 0.665, p_2 = 0.335, \lambda_1 = 0.05, \lambda_2 = 1, \lambda_3 = 0.1, \lambda_4 = 2, C_0 = 7000, C_1 = 1000, C_2 = 200, C_3 = 100, C_4 = 100 \)

Fig. 2: MTSF Versus Failure Rate

Fig. 3: Profit (P) Versus Availability When Demand is Not Less Than Production

Fig. 4: Profit (P) Versus Failure Rate (\( \lambda \)) for Different Values of Repair Rate (\( \alpha \))

Fig. 5: Profit (P) Versus Revenue Per Unit Time (\( C_0 \)) for Different Values of Cost (\( C_3 \))

Fig. 2 shows the behavior of MTSF with respect to failure rate. It can be concluded from the graph that MTSF decreases as failure rate increases.

Fig. 3 shows the behavior of Profit with respect to availability when demand is not less than production. It can be concluded from the graph that profit increases when availability of the system increases.

Fig. 4 shows that the behavior of Profit with respect to failure rate for different values of repair rate. It can be concluded from the graph, Profit decreases with increase in failure rate and increases for higher values of repair rate.

Fig. 5 reveals the behavior of Profit with respect to revenue per unit time (\( C_0 \)) for different values of cost per visit of the repairman (\( C_3 \)). It can be concluded from the graph that the profit get increases with the increase in values of \( C_0 \) and lower values for higher values of \( C_3 \). It is also observed from the graph that...
1. For $C_3=100$, the profit is positive or zero or negative according as $C_0 >$ or $= < 400$ and hence revenue per unit up time should be fixed not less than 400.

2. For $C_3=200$, the profit is positive or zero or negative according as $C_0 >$ or $= < 600$ and hence revenue per unit up time should be fixed not less than 600.

3. For $C_3=300$, the profit is positive or zero or negative according as $C_0 >$ or $= < 800$ and hence revenue per unit up time should be fixed not less than 800.

IX. Conclusion

This paper helps in analysing the reliability indices of a cable manufacturing plant for general energy system. It can be concluded from the graph that the profit gets increased with increase in the values of availability, but get decreased with the higher values of failure rate. From the cut-off points of the revenue per unit up time, the cost for visiting the repairman can be fixed. Also, various rates/revenue per unit up time/costs can be obtained which help in deciding the upper/lower acceptable values of rates/costs so that the system is cost-effective. Other Graphs can be plotted and then various conclusions can be drawn which will be very useful in taking various decisions beneficial to the company.

References


