Buckling Analysis of Carbon Nanotube Reinforced Polymer Composite Plates

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Abstract

Buckling behavior is an important consideration in the design of structural components. In the present investigation, a study of multi-scale analysis of the buckling of a Carbon Nanotube (CNT) reinforced polymer composite plate is presented. A micromechanics model based on the Mori-Tanaka method is developed by introducing straight CNTs aligned in one direction. An attempt is made to find the buckling behavior using classical plate theory. The effect of plate aspect ratio is investigated on the plate buckling. The study is conducted primarily with an intention of observing the effect of CNTs when used in structural applications such as plates.

Keywords

Multi Scale Analysis, Composite Plates, Carbon Nanotubes, Buckling

I. Introduction

The buckling load of a laminated plate depends on a variety of parameters like properties of reinforcement. Carbon Nanotubes (CNTs) with exceptional stiffness and strength have been regarded as ideal reinforcement of composite. Both experimental and theoretical studies have shown that CNTs have extraordinary mechanical and electrical properties [1]. By measuring the mechanical response CNTs under tension obtained the tensile strength of SWCNTs ranging from 13 to 52 Gpa [2]. Molecular dynamics and molecular mechanics calculations also predicted high tensile strengths of CNTs, much higher than carbon fibers and steels.

It is illustrated that the notable mechanical properties by CNTs have stimulated much interest in their use to reinforce advanced composites. It has been shown that CNTs elastic modulus and tensile strength are over 1 TPa and 150 GPa, respectively, which makes them many times stiffer and stronger than steel while being three to five times lighter. Moreover, the technical and economic feasibility of using CNTs in reinforcing polymer composites is also evaluated [5]. The load transfer mechanisms and effective moduli of the SWCNT reinforced composites, is investigated using a continuum model. Moreover, the effective longitudinal Young’s modulus, bulk modulus of the composite is obtained [6]. Based on different assumptions for displacement fields, different theories for plate analysis have been devised. These theories can be divided into three major categories, the individual layer theories, the Equivalent Single-Layer (ESL) theories, and the three-dimensional elasticity solution procedures. These categories are further divided into sub-theories by the introduction of different assumptions. For example, the second category includes the CLPT, the first-order and higher-order shear deformation theories (FSDT and HSDT). Effect of the aspect ratio on buckling of composite plate is studied and it is shown that effect of the different boundary conditions on the buckling load increases with increasing aspect ratio [7]. In the present study, the buckling behavior of composite plate reinforced by carbon nanotubes under uni-axial compressive load has been investigated by using analytical methods. The material properties of single walled carbon nanotubes are obtained using the Mori-Tanaka method. The effect of agglomeration of carbon nanotubes on the critical buckling load is investigated using analytical micromechanics methods.

II. Micromechanical Model

The micromechanical model involves the elastic behaviour of CNRP reinforced with aligned nanotubes which are straight and infinitely long. In the micromechanics model, the SWNTs are considered to be solid fibers with anisotropic material properties and the values of the elastic constants are taken from Popov et al [21]. The bonding at the nanotube–polymer interface is taken to be perfect. The composite is considered as transversely isotropic. The elastic behaviour of an elementary cell of the composite material can be expressed as

$$\sigma_{ij} = \begin{bmatrix} n & l & l & 0 & 0 & 0 \\ l & k + m & k - m & 0 & 0 & 0 \\ l & k - m & k + m & 0 & 0 & 0 \\ 0 & 0 & 0 & 2m & 0 & 0 \\ 0 & 0 & 0 & 0 & 2p & 0 \\ 0 & 0 & 0 & 0 & 0 & 2p \end{bmatrix} \begin{bmatrix} E_{11} \\ E_{22} \\ E_{33} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $k, l, m, n, p$ are the Hill’s elastic moduli; $n$ is the uni-axial tension modulus in the fibre direction (direction 1), $k$ is the plane-strain bulk modulus normal to the fibre direction, $l$ i the associated cross modulus, $m$ and $p$ are the shear moduli in planes normal and parallel to the fibre direction, respectively, as specified in J. Wuite and S. Adali [20]. A composite with a reinforcing phase volume fraction $\nu$, matrix Young’s modulus $E_m$ and matrix Poisson’s ratio $\nu_m$ is considered. Using the Mori–Tanaka method, the Hill’s elastic moduli are found to be

$$k = \frac{E_m[(1 + \nu_m)(1 - 2\nu_m)]}{(1 - \nu_m)(1 + \nu_m)}$$

$$l = \frac{E_m[(1 + \nu_m)(1 - 2\nu_m)]}{(1 - \nu_m)(1 + \nu_m)}$$

$$m = \frac{E_m[(1 + \nu_m)(1 + \nu_m)]}{(1 - \nu_m)(1 + \nu_m)}$$

$$\nu_m = \frac{E_m[(1 + \nu_m)(1 - 2\nu_m)]}{(1 - \nu_m)(1 + \nu_m)}$$

Based on different assumptions for displacement fields, different theories for plate analysis have been devised. These theories can be divided into three major categories, the individual layer theories, the Equivalent Single-Layer (ESL) theories, and the three-dimensional elasticity solution procedures. These categories are further divided into sub-theories by the introduction of different assumptions. For example, the second category includes the CLPT, the first-order and higher-order shear deformation theories (FSDT and HSDT). Effect of the aspect ratio on buckling of composite plate is studied and it is shown that effect of the different boundary conditions on the buckling load increases with increasing aspect ratio [7]. In the present study, the buckling behavior of composite plate reinforced by carbon nanotubes under uni-axial compressive load has been investigated by using analytical methods. The material properties of single walled carbon nanotubes are obtained using the Mori–Tanaka method. The effect of agglomeration of carbon nanotubes on the critical buckling load is investigated using analytical micromechanics methods.
where \(k_r, l_r, m_r, n_r\) and \(p_r\) are the Hill’s elastic moduli for the reinforcing phase. The expressions for the moduli of the CNRP as functions of the stiffness constants are determined for a unidirectional composite as follows:

\[
E_L = n - \frac{i^2}{k}, \quad E_T = \frac{4m(k_n - i^2)}{k_n - i^2 + mn}, \quad G_{LT} = 2p \quad \text{and} \quad \nu_{LT} = \frac{l}{2k} \tag{3}
\]

### III. Basic Equations

The equations used for the analytical solution of buckling of simply supported laminated beams are [19] as follows:

\[
M = bM_{xx}, \quad E_{xx} = \frac{12}{h^2D_{11}}, \quad l_{yy} = \frac{12}{h^2}, \quad l_{xy} = \frac{b}{l_{yy}D_{11}} \tag{4}
\]

\[
N_{cr} = \left(\frac{\pi^2}{12}\right)\frac{E_{xx}h^3}{a^2} \tag{5}
\]

The Classical Laminate Plate Theory is used to work out the analytical solution as it is simple to use for a moderate thick laminated plate. In the simply supported boundary conditions at four edges \((s - s - s - s)\), the critical axial buckling load for a uniaxial compression of a rectangular laminate as shown in Figure 1 is given by equation (6) [19].

![Fig. 1: Buckling of a Simply Supported Rectangular Plate Under Uni-axial Compression.](image)

\[
N_0(m, 1) = \frac{E_{xx}}{b} \left[ \frac{m^2D_{22}(\frac{2}{b})^2}{2} + \frac{(D_{22}+2D_{66})}{D_{22}} \right] + \frac{1}{m^2}\left(\frac{2}{b}\right)^2 \tag{6}
\]

The graphs are plotted considering Non-dimensionalised buckling load given in equation (7) on the ordinate.

\[
\overline{N}_0 = N_0b^2/(n^2D_{22}) \tag{7}
\]

### IV. Results and Analysis

The engineering constants for the unidirectional CNRPs are calculated using micromechanics equations (1) to (3). The polymer matrix is specified as polystyrene which has demonstrated good dispersion and a strong interfacial bond with nanotubes in experimental work [20]. The properties used in the present analysis are: The Young’s Modulus and Poisson’s ratio of Polystyrene are \(E_m = 1.9\) GPa, \(\nu_m = 0.3\).

A MATLAB program is developed to study the buckling behavior of beams and plates. The developed program takes care of variations of volume fractions of CNTs and stacking sequences. The nanotube radius is assumed to be 10 Å for all the cases otherwise mentioned for which the representative values of the elastic constants of SWCNTs are: \(n_r = 450\) GPa, \(k_r = 30\) GPa, \(m_r = p_r = 1\) GPa and \(l_r = 10\) GPa which are taken from the analytical results of [21].

To validate the developed program, a simply supported beam problem is considered whose results are available in the literature. The beam is subjected to concentrated load at the centre. The deflections computed for various stacking sequences using the developed program are found in good agreement with the published results [20].

### A. CASE I: Buckling of Beam

The buckling behaviour of a simply supported eight layered symmetric composite beam is studied with respect to various stacking sequences and fiber volume fractions.

![Fig. 2: Curves of Nondimensionalised Critical Buckling Load Plotted Against CNT Volume Fraction for Various Stacking Sequences for a Simply Supported Beam.](image)

From the above, it is observed that for the stacking sequence of \([0/45/-45/90]_s \& [0/90/0/90]_s\) the slope almost remains the same. Also for the stacking sequence of \([45/-45/45/-45]_s\) the slope is very small, which means as the percentage of reinforcement changes, there is no significant change in the critical buckling load.

![Fig. 3: Curves of Nondimensionalised Critical Buckling Load Plotted Against CNT Volume Fraction for Various Radii for a Simply Supported Beam.](image)
B. CASE II: Buckling of Plate

From the above graphs it is observed that there exist intersections of two modes corresponding to certain aspect ratios indicating for each of these aspect ratios there are two possible buckled mode shapes. For example, the stacking sequence [90/45/-45/0]s has an aspect ratio approximately 1, at which two buckled modes 1 and 2 exist.

From the above figs. 7 & 8, the observation is that the carbon nanotube radii and fiber volume fraction do not have any significant effect on the nondimensionalised buckling load for any stacking sequence.

Form the above figs. 7 & 8, the observation is that the carbon nanotube radii and fiber volume fraction do not have any significant effect on the nondimensionalised buckling load for any stacking sequence.
As the aspect ratio increases, there is a considerable increase in the non-dimensionalised buckling load. This is evident from the above fig. 9. Fig. 10 below indicates the variation of buckling load for various stacking sequences as the modulus ratio increases.

![Buckling Load Vs Modulus Ratio for CNT Radius 10Å, Cr=0.1, aspect ratio 1.5 & various stacking sequences](image)

**Fig. 10:** Curves of Nondimensionalised Uniaxial Critical Buckling Load Plotted Against Modulus Ratio for a Simply Supported Plate for Various Stacking Sequences.

**V. Conclusions**

The buckling of nanocomposite plates is examined by using micromechanics relations to determine the elastic constants in terms of nanotube volume fraction. It can be concluded that there exist intersections of two modes corresponding to certain aspect ratios indicating for each of these aspect ratios there are two possible buckled mode shapes. Also, the carbon nanotube radii and fiber volume fraction do not have any significant effect on possible buckled mode shapes. As the aspect ratio increases, there is a considerable increase in the non-dimensionalised buckling load.

**References**


[21] Popov VN, Dore VE, Balkanski M., “Elastic properties of