Kinematic Analysis of Lifting Booms System for Aerial Work Platform Based on Differential Forward Incremental PID controller

Haiyan He, Tianhong Luo, Xinxian Yin, Sunke Zhu, Jiayuan Luo

Abstract
By analyzing the kinematic characteristics of Lifting Booms System (LBS) of Aerial Work Platform (AWP) about its working process, this paper provided a differential forward incremental PID (DFI-PID) controller to improve its control accuracy. Based on operating principles and structural parameters of LBS, three mathematical models were established: the mechanical motion system of each lifting boom, the mechanical-hydraulic coupling system and the electro-hydraulic proportional system. Meanwhile, a model of LBS was built up based on the electro-hydraulic proportion of AMES. A joint simulation was conducted by integrating the differential forward incremental PID (DFI-PID) controller into AMES (working as a major simulation environment) through certain software interface. Then, a DFI-PID closed-loop controller was designed to analyze the effect of the PID controller on LBS. The result showed that compared with the conventional PID controller, the DFI-PID closed-loop controller had better steady and higher robustness towards command changes and load disturbance. Besides, the validity of joint simulations was also proved.

Keywords
Aerial Work Platform (AWP); Lifting Booms System (LBS); Kinematic Analysis; Closed-loop Control System; Differential Forward Incremental PID (DFI-PID) Controller

I. Introduction
As an emerging technological industry, AWP is an important branch in the engineering machinery field and is widely applied in shipbuilding, construction, municipal construction, fire control and port cargo etc [1-2]. Stability of LBS is an important factor to evaluate the success of AWP products and to ensure the safe operation of AWP as well. What’s more, the flexible structure of the folding-arm AWP makes it possible to surmount obstacles and reach the predetermined position, this can be applied to lower operating height AWP [3-4].

With the development of digital control technology, problems that could not be solved by analog PID controller before [5-6] are now easily done on a digital computer. Thus, based on current digital control technology, a series of control algorithm has been promoted to improve system quality and meet the needs of different control systems [7]. Therefore, in this paper, an algorithm of the DFI-PID controller was established, in which two advantages of incremental PID and differential forward PID was highlighted, respectively. Firstly, this algorithm is only related to the change of the most recent K power instead of accumulating, so that the faulty operations influence is minored and a better control effect can also be obtained by weighted processing [8]. Secondly, this method only has differential influence on the output rather than a given instruction [9]. Overall, the improved PID algorithm can be applied to situations where the instructions of lifting arms are frequently given in the process of adjusting the lifting positions. At this point, it is useful to avoid the excessive overshoot from instruction changes. Additionally, it also can avoid some tiny faulty operations that affect the control accuracy. Thus, the improved algorithm contents the needs of being continuous, stable, and fast dynamic responded in lifting and lowering process.

Based on current lifting booms control system (LBCS) technology, this paper employed the MATLAB/Simulink software to establish an integrated system model with the closed-loop controller. Meanwhile an integrated environment of the LBCS was built, with this integrated environment conducting combination to analyze of this system.

II. The Structure of the Lifting Booms System
The structure of the opening chain of series arm forms applied in the LBS was commonly used in industry, including the lower arm, the middle arm, upper arm, and a work platform. Each lifting arm and the link of the lower arm with the rotary table were hinged with the horizontal hinge pin. The hinge joint was provided with a special sliding bearing to reduce the resistance while the working arm rotating.

The concrete structure is shown in fig. 1:
III. Mathematical Models of the LBS

This section researched three parts of mathematical model for the LBS, such as mechanical motion for each lifting arm, the hydraulic coupling mechanical of the LBS and the electro-hydraulic system for the mechanical of the LBS.

A. Mathematical Model of Mechanical Motion for Each Lifting Arm

There provided three motion mathematical models, the lower arm, the middle arm and the upper arm as objects.

1. Motion Mathematical Model of the Lower Arm and the Upper Arm

\[ y^2 = a^2 + b^2 - 2ab \cos \theta \]  \hspace{1cm} (1)

Where \( y \) is total length of the lower arm (the upper arm) cylinder; \( a \) is the distance of the two hinge joints, one hinge joint is intersection for the oil cylinder of the lower arm and the rotary table, another hinge joint is intersection for the rotary table and the lower arm; \( b \) is the distance of the two hinge joints, one hinge joint is intersection for the rotary table and the lower arm, another hinge joint is intersection for the oil cylinder of the lower arm and the lower arm; \( \theta \) is the angle of the lower arm and rotary table. While, for the upper arm, \( a \) is the distance of the two hinge points, one hinge point is intersection for the upper arm cylinder and the middle arm, another hinge point is intersection for the middle arm and the upper arm; \( b \) is the distance of the two hinge points, one hinge point is intersection for the upper arm and the middle arm, another hinge point is intersection for the upper arm and the upper arm cylinder; \( \theta \) is the angle of the middle arm and the upper arm. Took the derivative of \( y \) and \( \theta \) with respect to time \( t \), then the equation is given as

\[ y' = f \theta' \] , where \( f \) represents the lower arm or the upper arm.

\[ f = \frac{ab \sin \theta}{\sqrt{a^2 + b^2 - 2ab \cos \theta}} \]  \hspace{1cm} (2)

II. Motion Mathematical Model of the Middle Arm

Fig. 3: Vector diagram of middle arm barycenter

\[ I_4, I_5 \] are the simplified connecting rods of the middle arm (In order to facilitate the analysis, the arm is divided into two sections), namely the rack; \( I_1 \) is the driver parts which is the distance of the two hinge points, one hinge point is intersection for the lower arm and the middle arm, another hinge point is intersection for the middle arm lower connecting rod and the lower arm; \( I_2, I_3 \) are the middle arm connecting rods of isometric; \( l_i \) is the total length of middle oil cylinder.

With the method of the reverse theory, namely that the middle arm and the lower arm were assumed to be frame and the driving link, respectively. Then the change of \( l_i (Y) \) was confirmed through the variation of angle of the driver and the frame. This equation can be defined via the complex vector method:

\[ A \sin \theta + B \cos \theta + C = 0 \]  \hspace{1cm} (3)

With \( A = 2l_3 I_3 \sin \theta_1 \), \( B = 2l_4 (I_4 \cos \theta_1 - l_4) \).

And \( C = I_5^2 - I_1^2 - I_2^2 - I_3^2 + 2l_1 l_4 \cos \theta_1 \).

Furthermore, the following relations can be achieved based on above equation formulas.

\[ \tan \theta / 2 = (A \pm \sqrt{A^2 + B^2 - C^2}) / (B - C) \]  \hspace{1cm} (4)

B. Mathematical Model of the Hydraulic Coupling and Mechanical of the LBS

Suppose that a scheme that considered the lifting arm was rotating at a speed of equiangular and the electro-hydraulic proportion servo valve was matching and symmetry. This application required \( x_1 = \theta \), \( x_2 = y \), \( x_3 = x_4 = y_4 = P_1 \), \( x_5 = p_2 \), hence, state space formula of nonlinear model was defined via flow continuity equation and force balance equation.

\[ x_1 = x_3 / f_1 \]
\[ x_2 = x_3 \]
\[ x_3 = \frac{(K / m)x_3 - (B / m)x_3 - (A_1 / m)x_3 - (A_2 / m)x_3}{(C_1 / V_1)x_3 - (C_2 / V_2)x_3} \]
\[ x_4 = \frac{(A_2 / V_1)x_3 + (C_2 / V_2)x_3 - x_4}{X_1} \]
\[ x_5 = \frac{(A_1 / V_1)x_3 + (C_1 / V_2)x_3 - X_1}{X_1} \]

Where, \( f_1 \) represents the lower arm, the middle arm or the upper arm; \( \theta \) is the rotation angle of the lifting arm; \( m \) is the equivalent quality of piston and load; \( B \) is the viscous damping coefficient of piston and load; \( K \) is the spring stiffness of load; \( A_1 \) and \( A_2 \) are effective area of non-rod cavity and rod cavity for the hydraulic cylinder, respectively; \( P_1 \) is the pressure of oil supply; \( P_2 \) is the return oil pressure, approximately pressure of 0; \( X_1 \) is the displacement of spool; \( W \) is the area gradient of throttle window for slide valve; \( C_1, C_2 \) are the flow coefficient; \( C_1 \) and \( C_2 \) are the inside and outside leakage coefficient of the hydraulic cylinder, respectively; \( P_1 \) and \( P_2 \) are the two cavity pressures of the hydraulic cylinder, respectively; \( V_{10} = V_{20} / \beta \), \( V_{10} \) and \( V_{20} \) are the initial volume of the non-rod cavity and rod cavity for the hydraulic cylinder, respectively; \( \beta \) is the modulus elasticity of oil bulk.

C. Mathematical Model of the Electro-Hydraulic System for the Mechanical of the LBS

1. Transfer Function of Amplifier

In order to discuss the formula accurately, the electrical links were regarded as general electric amplifier, namely the input and
the output were voltage and current. Also, it could be seen as a proportion link, the output performance of the amplifier was very high. Transfer function is defined as:

\[ W(s) = \frac{X(s)}{I(s)} = \frac{K_s}{s^2 + 2\xi\omega_n s + \omega_n^2} \]

(7)

Here: \( K_s \) is the proportional gain; \( \omega_n \) is the inherent frequency of proportional valve; \( \xi \) is the damping ratio.

3. Transfer Function of the Power Element

The hydraulic power units of each lifting arm were valve controlled asymmetrical cylinder. The follow type took consideration of the displacement variation, which cased by asymmetric cylinder area. Specifically, inertia load and external load were mainly taken into account, the transfer function is as shown:

\[ X(s) = \frac{A_s}{A_0} C \left[\frac{V}{T_b K_a} s + \delta \right] F_i + \frac{\nu_1 - 1}{1 + \nu_2} A_s p_i \]

(8)

Here: \( K_q \) is the flow gain; \( \nu_1 \) is the inherent frequency of hydraulic; \( \nu_2 \) is the hydraulic damping ratio; \( C \) is the variation coefficient of equivalent area for the load flow; \( T \) is the variation coefficient of elastic modulus for the effective bulk; \( \beta_1 \) is the oil tanks area; \( K_{ce} \) is the oil pressure; \( A_p \) is the equivalent area of the load flow; \( P_s \) is the oil pressure; \( \beta_2 \) is the modulus of elasticity for oil.

4. The Principle of the DFI-PID Controller Algorithm

Compared with conventional PID controller, the DFI-PID controller combined advantages of the differential forward PID and the incremental PID. The essence of the differential forward PID was early operation; this method was only to conduct the output increment. The control formula of incremental is obtained:

\[ u(k) = u(k-1) + K_p e(k-1) + K_i \sum_{i=0}^{k-1} e(i) + K_d (e(k) - e(k-1)) + u_0 \]

(11)

Here, \( u_0 \) is the based value of output controller when \( K=0 \); \( u(k) \) is the output value of controller for \( k \) th sampling time; \( K_p \) is proportion amplifying coefficient; \( K_i \) is integral amplifying coefficient, \( K_d = \frac{K_T}{T} \); \( T \) is differential time of oil. Therefore, if analog parameters \( K_p \), \( T \) and \( T_d \) of the PID controller were available, then in a shortly sampling time, the parameters of \( q_p \), \( q_i \) and \( q_d \) could be calculated out with \( K_p \), \( T \) and \( T_d \).

The AWP is only performing the differential for output \( y(t) \) of hydraulic cylinder piston rod for each lifting arm; the differential output signal contains controlled parameters and the change of the.
rate. The output is worked as a measured value putting into the proportional integral controller, as a result to strengthen the system to overcome the overshoot effect, thus, compensating the process lag to improve the control quality. This method is only related to the change of the nearest K power rather than accumulating, so the missing movements have minor influence, moreover, it is easy to have a good control effect through weighted processing [14]. Although, piston rod displacement for a desired value changes, there has little error actions and the output won't change substantially. In addition, because of the controlled quantity has few mutations, even if the given value is to alter, the charged quantity is slowly to change, thereby it is no longer result in mutant differential.

IV. Dynamic Characteristics Analysis of the LBS

A. Models and Setting Parameters

Under the environment of AMEsim with the principle of the lifting arm electro-hydraulic system and MATLAB / Simulink modeling, various kinds of modules of AMEsim model could be got from the Planar mechanical module library and the Mechanical module library. Finally, either with MATLAB / Simulink interface technology, AMEsim played a major role in simulation environment and built up the whole simulation model of the lifting arm system [15-16]. For the limits of the software model library, some sections couldn’t be completely copied from the prototype element, so that some of them were replaced, but all of them observed the principle of the system characteristic unchanged. The model is as shown in fig. 5.

Fig. 5: Simulation Model of Lifting System

In fig. 5, Body1 is the lower arm, Body2 is lower connecting rod of the middle arm, Body3 is upper connecting rod of the middle arm, Body4 is the middle arm, Body5 is the upper arm, Body6 is leveling on upper connecting rod, Body7 is leveling on lower connecting rod, Body8 is working bucket.

Denote the formula parameters by actual length and the function of (2) can be simplified, as given below:

\[
(16)
\]

where \( t \) represents the lower arm.

According to the initial installation of the middle arm, determined the sign in the formula of (1) was negative. With the aid of the triangle cosine formula and actual structure parameters, a formula for \( Y \) and \( \theta_1 \) can be defined:

\[
y = -\frac{350250\sin q}{140100\cos q - 2212801}
\]

That is the relation formula of the piston rod displacement(y) with angle \( \theta_1 \):

\[
y = \frac{1}{125} \sqrt{8761 - 10480 \cos(2 \arctan(\frac{650 - \sqrt{362 - 50\cos 20 - 312\cos^2 0}}{6\cos 0 - 6}))} - 1.326 (17)
\]

Took the derivative of \( y \) with respect to time \( t \), then the equation is given as \( y' = f'Y \), where \( t \) represents the middle arm.

\[
y' = \left\{ 0.85 \cdot \frac{650}{\cos 0} \left( -\cos 0 \cdot 12.5 \sin 20 - 11 - 2 \arctan \left( \frac{650 \cdot \cos 0}{9} \right) \right) \right\} \left( -\frac{3}{8} \sin 0 + 10 \sin 0 \right) - \left( 18761 - 10480 \cos(2 \arctan(\frac{650 - \sqrt{362 - 50\cos 20 - 312\cos^2 0}}{6\cos 0 - 6})) \right) \] (19)
Here, \( a = \sqrt{78 + 25 \sin^2 \theta - 78 \cos \theta} \)

Denoted the formula parameters by actual length of the upper arm and the function of (2) can be simplified, as given below:

\[
f_I = -\frac{102542 \sin \theta}{\sqrt{410168 \cos \theta - 810857}}
\]  

Selecting submodel system is very important for obtaining the correct simulation results. In the actual situation and under the mode of submodel, according to the actual type of each element, submodel of corresponding element could be selected in the drop-down list. Besides, simulation parameters were set up, as shown in Table 1.

![Bode Diagram of Lower Arm Barycenter System](image1)

**Fig. 6:** Bode Diagram of Lower Arm Barycenter System

![Bode Diagram of Middle Arm Barycenter System](image2)

**Fig. 7:** Bode Diagram of Middle Arm Barycenter System

![Bode Diagram of Upper Arm Barycenter System](image3)

**Fig. 8:** Bode Diagram of Upper Arm Barycenter System

### B. Testing and Analyzing the LBS

#### 1. Steady State Analysis of the Lifting Arm System

When simulation model of the system was established and the corresponding initial parameters were set, steady-state simulation of the lifting arm electro-hydraulic system could be in progress. Bode diagram of the LBS is as shown in fig. 6, 7 and 8. According to the control theory, quite stable conditions of the system are as shown:

- Amplitude margin \( K_a \geq 6 \text{dB} \)
- Phase margin \( \gamma = 30^\circ \sim 60^\circ \).

### Table 1: Simulation Parameters of the System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Numerical value</th>
<th>Company</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity of hydraulic oil</td>
<td>( 5.1 \times 10^{-2} ) Pa.s</td>
<td></td>
</tr>
<tr>
<td>Medium density of hydraulic oil</td>
<td>850 kg/m(^3)</td>
<td></td>
</tr>
<tr>
<td>Bulk modulus of hydraulic oil</td>
<td>1700 MPa</td>
<td></td>
</tr>
<tr>
<td>Temperature reference of hydraulic oil</td>
<td>40 ( ^\circ \text{C} )</td>
<td></td>
</tr>
<tr>
<td>Maximum displacement of main hydraulic pump</td>
<td>70 ml/r</td>
<td></td>
</tr>
<tr>
<td>Rated engine speed</td>
<td>1500 r/min</td>
<td></td>
</tr>
<tr>
<td>Pressure of relief valve ( e )</td>
<td>21 MPa</td>
<td></td>
</tr>
<tr>
<td>The diameter of piston ( L_{1p} )</td>
<td>100 mm</td>
<td></td>
</tr>
<tr>
<td>The diameter of piston rod ( L_{1p} )</td>
<td>50 mm</td>
<td></td>
</tr>
<tr>
<td>The length of the piston rod ( L_{1p} )</td>
<td>0.52 m</td>
<td></td>
</tr>
<tr>
<td>( K_f )</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>( K_i )</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>( K_d )</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>The diameter of piston ( L_{2p} )</td>
<td>113 mm</td>
<td></td>
</tr>
<tr>
<td>The diameter of piston rod ( L_{2p} )</td>
<td>56 mm</td>
<td></td>
</tr>
<tr>
<td>The length of the piston rod ( L_{2p} )</td>
<td>0.906 m</td>
<td></td>
</tr>
<tr>
<td>( K_f )</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>( K_i )</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>( K_d )</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>The diameter of piston ( L_{3p} )</td>
<td>63 mm</td>
<td></td>
</tr>
<tr>
<td>The diameter of piston rod ( L_{3p} )</td>
<td>35 mm</td>
<td></td>
</tr>
<tr>
<td>The length of the piston rod ( L_{3p} )</td>
<td>0.33 m</td>
<td></td>
</tr>
<tr>
<td>( K_f )</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>( K_i )</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>( K_d )</td>
<td>0.03</td>
<td></td>
</tr>
</tbody>
</table>
In this section, the results of the steady state analysis of the electro-hydraulic system for each arm were achieved from the graph of fig. 6, 7 and 8, as shown in Table 2. The tests reported above showed that each LBS has better stability.

Table 2: The Results of Steady-State Analysis

<table>
<thead>
<tr>
<th></th>
<th>Amplitude margin (db)</th>
<th>Phase margin (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The lower arm system</td>
<td>7.9</td>
<td>42.9</td>
</tr>
<tr>
<td>The middle arm system</td>
<td>7.13</td>
<td>55.6</td>
</tr>
<tr>
<td>The upper arm system</td>
<td>12.1</td>
<td>35.6</td>
</tr>
</tbody>
</table>

2. Dynamics Analysis of the LBS

When the LBS is working, lifting sequence and displacement of each lifting arms are different. In order to research it conveniently, this research took an example of the highest position and the operator used the most commonly operation sequence to analysis the result. What’s more, setting up simulation running time of 120s, step 0.01s. The analysis results of the simulation are shown as follows.

(a). The Lower Arm

In order to improve stability and reliability of the system, the DFI-PID controller algorithm was employed. The AMESim system with the aid of the batch processing, optimal parameters of the PID system was obtained. Furthermore, compared with the traditional hydraulic model (conventional PID controller) and draw the comparison chart of the entrance flow for the hydraulic cylinder, as shown in fig. 9.

Fig. 9: The Velocity of Lower Arm Barycenter

In fig. 9, barycentric velocity of the lower arm in the vertical direction can be quickly reached at a steady value without overshoot, this occurs due to the using of the DFI-PID. The dynamic characteristic of the system was improved greatly.

(b). The Middle Arm

As the same with the lower arm, in order to improve the dynamic characteristics, the DFI-PID controller was applied. The same method was used to get the optimal parameters of the PID system, compared with the traditional hydraulic model (conventional PID controller) and draw the comparison diagram of the entrance flow for hydraulic cylinder, as shown in fig. 10.

Fig. 10: The Velocity of Middle Arm Barycenter

From the graph of fig. 10, velocity in the vertical direction of the middle arm can reach at a steady condition rapidly with no overshoot. This great phenomenon was obtained by DFI-PID, much of the stability and reliability for the system were improved.

(c). The Upper Arm

Simulation analysis of the upper arm was identical with the middle arm. The AMESim system with the aid of the batch processing, optimal parameters of the PID system was obtained. The comparison chart of the entrance flow for the hydraulic cylinder (see fig. 11) was achieved by comparing with DFI-PID controller and the conventional PID controller.

Fig. 11: The Velocity of Upper Arm Barycenter

Fig. 11 shows that barycentric velocity of the upper arm in the vertical direction can be quickly reached at a steady value without an overstrike. This was achieved by DFI-PID controller, the dynamic characteristic of the system was improved greatly.

(d). Working Platform

In harmony with the method of the lifting booms analysis, with the algorithm of DFI-PID controller, then compared with the traditional hydraulic model (the conventional PID controller) and finished the contrast diagram of barycentric velocity in the vertical direction for the operating platform, as shown in fig. 12.

Fig. 12: The Velocity of Working Platform Barycenter

In harmony with the method of the lifting booms analysis, with the algorithm of DFI-PID controller, then compared with the traditional hydraulic model (the conventional PID controller) and finished the contrast diagram of barycentric velocity in the vertical direction for the operating platform, as shown in fig. 12.
Fig. 12: The Velocity of Working Platform

From the graph of fig. 12, the speed of operation platform in the vertical direction is running smoothly without jitter. Stability and reliability of the proposed system had been improved obviously.

V. Conclusion

This paper analyzed the operational mechanical of the AWP-LBS, based on which mathematical model of lifting system, electromechanical coupling model and transfer function of electro-hydraulic system were established. By taking advantages of simulation analysis software in MATLAB and AMES and the interface technology, an integrated simulation platform of the LBS was built to make united simulation realized. The results show that the system simulation model is effective and the DFI-PID controller can achieve better dynamic characteristics. The major aim of this paper is to research on the DFI-PID controller, the results are as follows:

1. Efficiency of the design can be improved by building a simulation model with AMES and modifying the design parameters instead of the physical parameters debugging.
2. Steady-state analysis of lifting booms for the upper arm, the middle arm and the lower arm (with Bode diagrams) shows that electro-hydraulic system of each arm is stable.
3. DFI-PID controller can perform better in vertically changing the speed of each arm smoothly and stably than conventional PID controller. Thus, good dynamic and static characteristics are obtained, without overshoot and oscillation.
4. Practical function of the promoted DFI-PID in this paper has been proved through this research and the MATLAB / Simulink and AMESim are tested to be valid and reliable, which puts lights on the future research and design on the AWP.

References

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