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Accurate Prediction of Wet Multiple Disc Brake Noise Used Complex Mode Method

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Abstract
This paper proposed an analysis modal and a complex-eigenvalue modal based on algorithm of wet multiple disc brake noise. The damping and stiffness between the friction plate and the dual steel plate were introduced on account of the small relative displacement of the friction plate and the dual steel plate due to time delay, otherwise, the friction contact relations of braking component and their effect also take into consideration, this kind of modal solve the problem that how to predict brake noise more accurately during braking, which could proved by the finite element model and the research has vital importance to practical engineering.

Keywords
Wet Multiple Disc Brakes, Complex Eigenvalue, Brake Noise

I. Introduction
In the past several decades years, the scholar have been studied the brake noise, initially scholars used the linear equation to predict noise tendency, J.Kang [1-2] proposed far-reaching noise mechanical which through the study of brake noise from linear noise model. However, the linear analysis model can't explain clearly the cause of the deviation from the equilibrium state of the nonlinear brake noise. So more and more scholars analyze the nonlinear time-domain of brake noise through finite element software [3]. AbuBakar and Ouyang [4-6] found that in the whole brake system, there are plenty of nonlinear factors including stiffness, friction coefficient , load, velocity, temperature and the complex relationship of each other. J.Kang[7] putted forward double-mode splitting model to study the nonlinear characteristic of brake noise during braking , this model was also mentioned in the reference [6]. However, this approach ignores the effects of friction plate rotation and radial friction force, which cannot put forward the essential characteristics of the noise behavior as Y.Hu and J.D. Fieldhouse [8-9] mentioned in the acoustic wave motion. With the development of science and technology, holographic technology has also been applied to the brake disc vibration model of surface visualization, there have been scholars proposed the brake noise model of pure traveling wave based on this technology. Reeves [10] putted forward the brake noise model about composite wave modal which consisting of several modal waves and analyzed its nonlinear model, obtaining solution of closed wave model. Because of the complexity of the derivation, the brake disc lateral double mode is rarely mentioned in other nonlinear brake noise model.

Many papers focusing on brake noise, but rarely mentioned the friction between the friction plate and dual steel sheet in the above models, which will directly affect the accuracy of the results. This paper will focus on the study of nonlinear brake noise model; the damping and stiffness between the brake disc and the friction plate were introduced based on the consideration of the relative displacement induced by the friction plate and dual steel plates, the nonlinear contact kinematics analysis is used to analysis the brake noise, proposed wet brake noise analysis model and complex eigenvalue algorithm.

II. Mechanism and Method of Brake Noise

A. Theory of Brake Noise
Friction torque force produced by the friction pair make the vehicle braking, physical contact is formed between the friction lining and brake wheel, friction coefficient changes with the relative speed of friction pairs; the friction consumption of kinetic energy, part of which is converted to heat, another part of the conversion for sound, causing the brake noise. Many factors influence the braking noise; therefore, the brake noise generation mechanism has no unified theory. Scholars are mainly studied in these three aspects : friction characteristics, geometric characteristics and structure coupling. The advantages and disadvantages of three theories as shown in fig. 1. The most authoritative theory is stick-slip friction characteristics which belongs to self-excited vibration theory, the theory explains that the friction coefficient decreases with the sliding speed in a certain range, resulting in the stick-slip phenomenon, which may led to divergent vibration and noise when the damping of the system is small [11].

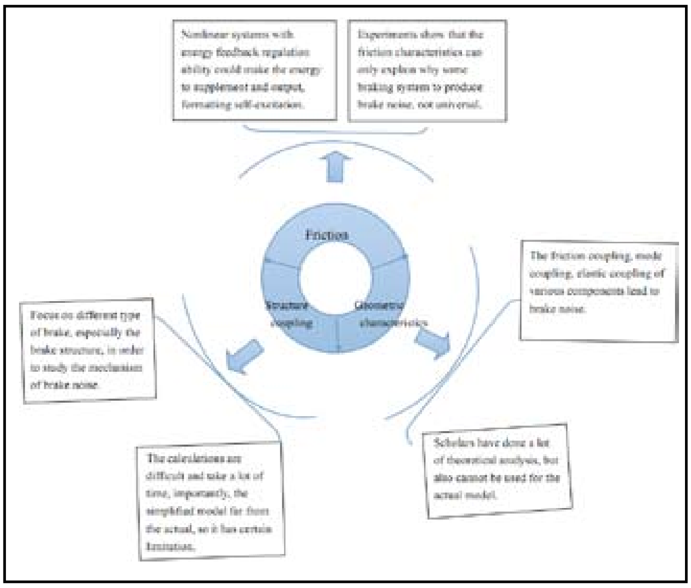

Fig. 1: The Mechanism of Brake Noise

B. The Method of Brake Noise Based on Complex Mode
There are mainly two kinds of methods to research brake noise: transient dynamic analysis method and complex eigenvalue method. Transient dynamic analysis method considered the nonlinear factors existing in the process of braking, namely the nonlinear transient process from the initial state to steady state, but it has a long calculation time, otherwise, the time domain solution needed a large amount of space and calculated data is not easy to modify, high frequency mode is easy to decay in the process of solving.

The complex eigenvalue method can be simulated the different structure, different types, different materials and different usage conditions of brake, Analyzing the characteristics in the vicinity of the equilibrium point to explore the influence on brake noise of inherent characteristics and the coupling characteristics of parts . The system is considered to be linearization in process, the occurrence tendency of brake squeal was judged according to the real part of characteristic roots of the complex eigenvalue, when eigenvalues of real part is less than zero, the system is in the stable state. When the eigenvalue is greater than zero, the system is in an unstable state, appearing the trend of brake noise. The complex relationship between the eigenvalue and the brake noise as shown in fig. 2:

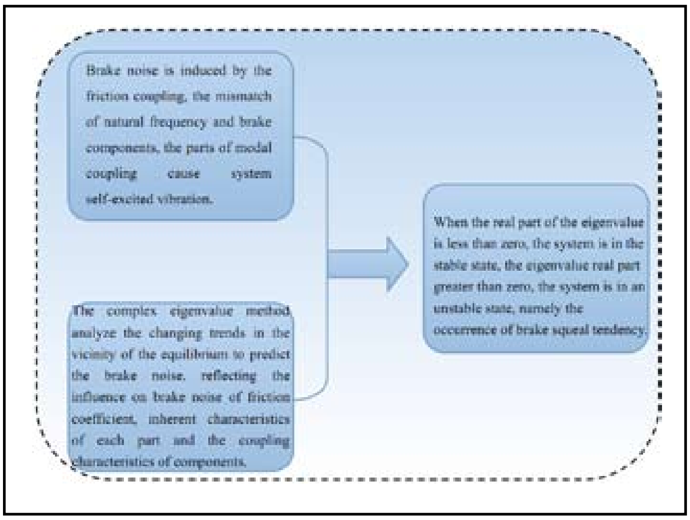

Fig. 2: Relationship Between Eigenvalue and Brake Noise

III. The Derivation of the Motion Equations

In order to improve the reliability of analysis and the accuracy of the prediction model, the damping and stiffness of the brake disc and the friction plate,relative displacement of the friction plate and the dual steel plates ,even the friction force induced by relative displacement are take into consideration. Figure 3 is structure diagram of the brake, the friction plate rotates at constant speed, where XYZ represents the coordinates of the friction plate, xyz represents the coordinate of the dual steel plates, this paper study the system which composed by the friction lining and two dual steel plates.

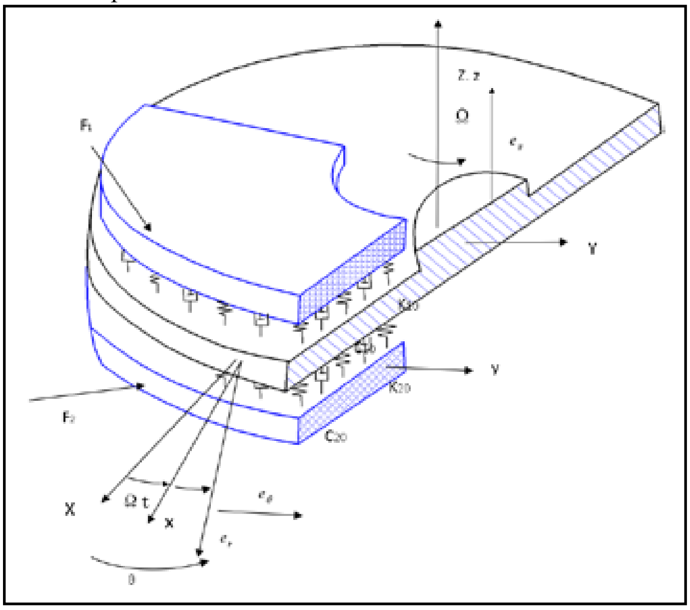

Fig. 3: A Simplified Model of the Wet Brake

The displacement vector of friction plate at a node X can be written as:

$$\tilde{u}_r(\tilde{X},t)e_r + \tilde{u}_\theta(\tilde{X},t)e_\theta + \tilde{u}_z(\tilde{X},t)e_z = \tilde{U}(\tilde{X},t) \quad (1)$$

At the same time, the displacement vector of the dual steel plates is:

$$u_r^{F_i}(X,t)e_r + u_\theta^{F_i}(X,t)e_\theta + u_z^{F_i}(X,t)e_z = U^{F_i}(X,t), i=1,2 \quad (2)$$

In the process of braking, the displacement vector of friction plate changed with the initial position.

$$r(\tilde{X},t) = \tilde{X} + \tilde{U}(\tilde{X},t) \quad (3)$$

$$\tilde{X} = re_r + ze_z$$

The initial coordinates were converted to the reference coordinate, for the time integration, friction plate velocity is:

$$V(X,t) = \left[\frac{dr(\tilde{X},t)}{dt}\right]_{XYZ} = \left[\frac{d\tilde{u}(\tilde{X},t)}{dt}\right]_{xyz} + (\Omega k)\times r(X,t) \quad (4)$$

Rotating coordinate was transform to the datum coordinate:

$$\left[\frac{d\tilde{u}(\tilde{X},t)}{dt}\right]_{xyz} = \frac{\partial u(X,t)}{\partial t} + \Omega\frac{\partial u(X,t)}{\partial\theta} \quad (5)$$

In order to more directly reflect the change in the process of rotating, the coordinate of the dual steel plates are also replaced by the datum coordinate:

$$V^{F_i}(X^{F_i},t) = \frac{\partial u^{F_i}(X^{F_i},t)}{\partial t} \quad (6)$$

According to Kulun's law of friction, the friction force of contact surface S can be expressed as:

$$F(X_s^{F_i},t) = -up^{F_i}d^{F_i}, i=1,2 \quad (7)$$

The direction velocity unit is d^{F_i} , which could be derived from the formula of the slip velocity formula:

$$d^{F_i} = \frac{\{V(X_s^{F_i},t) - V^{F_i}(X_s^{F_i},t)\}}{|V(X_s^{F_i},t) - V^{F_i}(X_s^{F_i},t)|}, i=1,2$$

When the effect of P0 on the contact surface produced a static load, static load in general can be expressed as:

$$p^{F_i} = p_0 + k_s\left[\{u(X_s^{F_i},t)\text{-}u^{F_i}(X_s^{F_i},t)\}e_z\right] \quad (8)$$

Friction characteristics curve is a function of stick slip velocity which is used to determine whether the stick-slip motion will cause vibration [11].The stick- slip velocity can be expressed as:

$$u(r,t) = \left\{u_k + (u_s - u_k)e^{-\beta_1|V(X_s^{F_i},t)-V^{F_i}(X_s^{F_i},t)|}\right\}$$
$$\times\left(1 - e^{-\beta_2|V(X_s^{F_i},t)-V^{F_i}(X_s^{F_i},t)|}\right) \quad (9)$$

Friction force induced by stick-slip velocity :

$$p^{F_i} = \frac{1}{2}(p^{F_i} + |p^{F_i}|) \quad (10)$$

The total virtual work of friction force is:

$$\delta W = \int_{A_s^{F_i}} \{F(X_s^{F_i})\delta U(X_s^{F_i}) - F(X_s^{F_i})\delta U^{F_i}(X_s^{F_i})\}dA, i=1,2 \quad (11)$$

The displacement vector of the friction surface is:

$$U(X,t) = \sum_{n=1}^{n}\left[\{\varphi_{r(n)}(X)e_r + \varphi_{\theta(n)}(X)e_\theta + \varphi_{z(n)}(X)e_z\}q_n(t)\right] \quad (12)$$

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$$U^{F_i}(X,t)=\sum_{n=1}^{n}\left[\left\{\varphi^{F_i}_{r(n)}(X)e_r+\varphi^{F_i}_{\theta(n)}(X)e_\theta+\varphi^{F_i}_{z(n)}(X)e_z\right\}q^{F_i}_n(t)\right],i=1,2 \quad (13)$$

The equations of motion can be expressed as:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{a}_m}\right]-\frac{\partial L}{\partial a_m}=\sum_{n=1}^{N_F+N_{2D}}F_{mn}(a_n) \quad (14)$$

Where: $\Delta U=T-U-U_1$

$m=1,2...(\mathrm{N}_F+2\,\mathrm{N}_D)$

$\mathrm{a}=\left\{q^{F_1}\quad q^d\quad q^{F_2}\right\}^T=\left\{a_1\quad a_2\quad ...\quad a_{N_F+N_{2D}}\right\}$

The contact strain can be expressed as:

$$U_1=\frac{k_s}{2}\int_{A_s^{F_i}}\left[\left\{U(X_s^{F_i},t)-U^{F_i}(X_s^{F_i},t)\right\}K\right]dA,i=1,2 \quad (15)$$

The total friction F_{mn} can be derived through the discrete friction power.

$$\delta W=\sum_{n=1}^{N_F+2N_D}F_{mn}(a_n).\delta a_m \quad (16)$$

A general form of linear system of equations is:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K-K_f]\{u\}=\{0\} \quad (17)$$

The left side of equality on both sides multiply $[M]^{-1}$,

$\{\ddot{u}\}+[M]^{-1}[C]\{\dot{u}\}+[M]^{-1}[K-K_f]\{u\}=\{0\}$

Where:

$$\begin{Bmatrix}\ddot{u}\\ \dot{u}\end{Bmatrix}+\begin{bmatrix}-MC & -M^{-1}(K-K_f)\\ I & 0\end{bmatrix}\begin{Bmatrix}\dot{u}\\ u\end{Bmatrix}=\{0\}$$

The form of the root is

$\lambda_j=\gamma_j\pm i\omega_j,(\mathrm{j}=1,2,3...,\mathrm{n})$

According to the linear system stability can be judged: if $r_j<0$, the system is in stable state; if $r_j=0$ the system is in boundary stable state; if $r_j>0$ the system is not stable, the there is tendency of brake noise.

Analyzed the stress of the friction plate and the dual steel plates, written in matrix form:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K-K_f]\{u\}=\left[F_{mn}\right] \quad (18)$$

Brake noise frequency determined by the natural frequency of the unstable modes. Bring the parameters of wet brake into Jacobin matrix; calculate the eigenvalue of the system using the solver, judge the stability of brake process by the real part of value .Table 1 is parameters of wet brake.

Table 1: Parameters of Wet Brake

Parameter	Value	Parameter	Value
$m^{F_1}=m^{F_2}$	0.3Kg	$c_{10}=c_{20}$	6N·s/m
m_d	0.5Kg	$k_{2x}=k_{2y}$	9600N/s
$k_{10}=k_{20}$	13000 N/s	v_0	2.77m/s

Bring the parameters into Jacobin matrix, obtained 12 characteristic values, can be seen in Table 2.

Table 2: The Eigenvalues of the Jacobin Matrix

Number	Eigenvalue
1	-8.50 + 190.12i
2	-8.50 - 190.12i
3	-3.13+792.04i
4	-3.13-792.04i
5	-7.76+267.45i
6	-7.76-267.45i
7	3.98+267.05i
8	3.98-267.05i
9	-9.06+164.78i
10	-9.06-164.78i
11	-2.64+352.58i
12	-2.64-352.58i

From the above results, it can be seen that the characteristic value are divided into two parts, a real part and an imaginary part. Imaginary part always appeared in pairs. When the real part is less than zero, the system is in a stable state. When the real part is greater than zero, the system is in a flux state. Since there are two characteristic values of the real part and greater than zero, which can predict the system is unstable, the unstable frequency is 276.05Hz.

IV. Finite Element Analysis

A. The Establishment of Finite Element Model

Since friction contact is a time-variant dynamic process, the contact relationships between the friction linings and steel plates should be taken into consideration. The friction plate and steel disc contact include normal and tangential contact, the normal contact has little impact on brake noise [12], so it could be neglected in the research. Tangential contact composed by dynamic friction and static friction. Get the contact state of the friction pairs through static analysis, and analyze the mode of system on this basis, namely prestressed modal analysis, finite element model as shown in fig. 4.

Static analysis: Fixed constraints are imposed on radial friction plates while the axial and tangential movements are free and the speed is 3 rad/s. Fixed constraints are also imposed on dual steel plates in radial and tangential direction while the axial movement is free. The 4-Mpa pressure is applied on both sides. In order to facilitate the convergence of analysis and solution, it selected a non-symmetric contact and the enhanced Lagrange algorithm.

Prestressed modal analysis: Based on the static analysis, Static analysis results are extracted from boundary conditions directly. Because of the introduction of asymmetric friction contact, extracted the first 30-order modes.

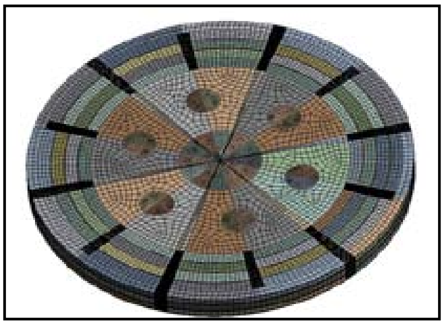

Fig. 4: The Finite Element Model

B. Material Properties of Wet Brake

The friction plate and dual steel sheet selection of 65Mn steel which was regarded as a linear material in the analysis, and give proper young's modulus .The density is7228 kg. m^{-3} , Poisson's ratio is 0.3, considering the special working environment, select the transmission oil as brake fluid, the temperature is 40.℃,the density is 852 kg.m^{-3} , the specific heat is 2131 J. (kg.℃), the coefficient of thermal conductivity is 0.138W(m.℃), the viscosity is52.18 mm^2 .s^{-1}, other parts of the material as shown in Table 3.

Table 3: Materials Properties of Wet Brake Parts

Parts	Young's modulus/Mpa	Density/ $kg \cdot m^{-3}$	Poisson's ratio	Material name
Dual steel plates	175000	7228	0.3	65Mn steel
Friction linings	175000	7228	0.3	65Mn steel
Friction materials	1500	1450	0.25	Paper based friction material without asbestos

C. Results of Finite Element Analysis

Test results indicated the instability frequency of wet brake system is generally about 100 to 300Hz , which belongs to the low frequency vibration noise, the first 30 modes of the extraction, as shown in fig. 5. Unstable frequency will appear an imaginary part, two mutually opposite number of the real part, the upper portion of the diagram point represent real part instability frequency value.

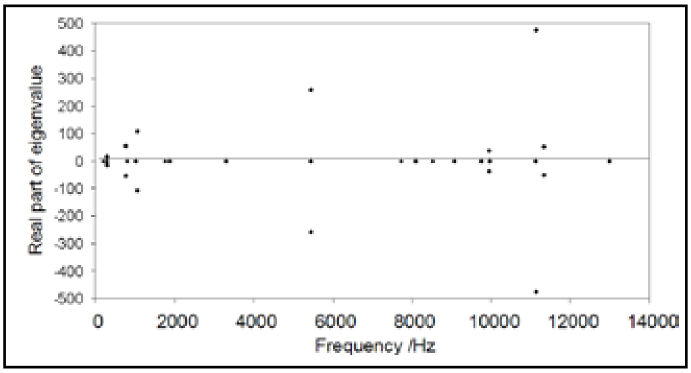

Fig 5: The Complex Eigenvalue of Braking System

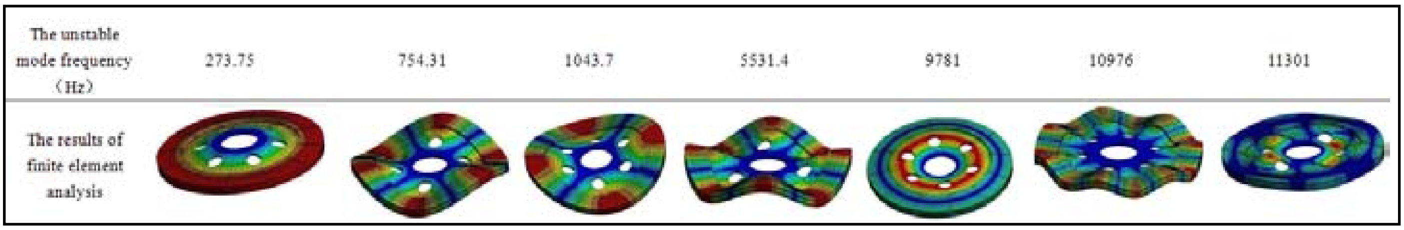

Fig. 6: The Unstable Mode and Frequency

1. Eigenvalues come in pairs from the extracted thirty complex modal, as shown in Figure 5, the value of the imaginary part is the same, the value of the real part is opposite. There are 7 orders belong to the unstable order, but only one order belongs to the low frequency noise, the rest belong to the high frequency noise.
2. From fig. 7 it can be concluded that the unstable frequency values are 273.75Hz, 754.31Hz, 1043.7Hz, 5531.4Hz, 9781Hz, 10976Hz, and 11301 Hz. the finite element modal analysis of instability also can be seen in Figure 7.
3. Numerical calculation of unstable frequency is 267.05Hz, finite element analysis obtained the unstable frequency was 286.75Hz, the results of numerical analysis and finite element simulation are closed, means the description of the method is indeed improves the prediction precision.

V. Conclusion

The damping and stiffness of the friction plate and the dual steel plate, relative displacement of the friction plate and the dual steel plates, even the friction force induced by relative displacement are taking into consideration. The nonlinear wet brake mathematical model was stetted up, and the calculation formula of the corresponding theory was deduced, and the model was verified by finite element method, the results show that the model can improve the prediction accuracy.

The brake noise problem is connected with many parameters such as temperature, humidity, speed, contact state, wear and other related physical behavior, the present analysis does not include these parameters, the research in the future will consider the impact of these factors on the brake noise, resulting to more accurately predict the brake noise.

Nomenclature

$\tilde{u}_r$、$\tilde{u}_\theta$、$\tilde{u}_z$: The displacement vector of friction plate in r、θ、z direction

$u_r^{F_i}$、$u_\theta^{F_i}$、$u_z^{F_i}$: The displacement vector of dual steel plate in r、θ、z direction

$\{\varphi_{r(n)}, \varphi_{\theta(n)}, \varphi_{z(n)}\}$ Modal matrix of friction plate

$\{\varphi_{r(n)}^{F_i}, \varphi_{\theta(n)}^{F_i}, \varphi_{z(n)}^{F_i}\}$: Modal matrix of dual steel plate.

$\tilde{U}(\tilde{X}, t)$: The displacement vector of the friction plate

$U^{F_i}(X, t)$: The displacement vector of dual steel plate

$r(\tilde{X}, t)$: The change of displacement

$\tilde{X}$: The initial displacement

$V(X, t)$: The velocity vector of the friction plate

$V^{F_i}(X^{F_i}, t)$: The velocity vector of dual steel plate

$F(X_s^{F_i}, t)$: The friction force of contact surface

u : The friction coefficient

p^{F_i} : Dynamic load

d^{F_i} : Direction vector

p_0 : Static load

a_m : The coordinates of M modal

m : M modal

N_F : The mode of friction plate

N_D : The mode of dual steel plate

U : Strain energy

U_1 : Kinetic energy

F_{mn} : The total force of friction

M : The quality of the system matrix
C : The damping matrix
K : The stiffness matrix

K_f : Friction stiffness matrix

u : Displacement vector of System

ω_j: Natural frequency of J mode

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