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Calculated Performance of Misaligned Finite Circular Journal Bearing

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Abstract
Misalignment of the shaft in the bearing is created because of bending of the flexible shaft. Bending takes place due to unbalance present in the rotor mounted on the shaft. Reynolds equation becomes complicated due to presence of small skew angles. Also boundary conditions change and the mathematical analysis becomes comparatively difficult. The Reynolds equation is solved with the help of Galarkins method. It is found that the magnitude of non dimensional pressure increases in circumferential direction. Load carrying capacity increases with eccentricity ratio, and attitude angle decreases with eccentricity ratio.

Keywords
Misalignment, Reynolds Equation, Galerkin's Method, Load Capacity, Attitude Angle

I. Introduction
In this work Reynolds equation is solved with the help of Galerkin's method. Some researchers have observed serious effect of even small degree of misalignment on the bearing performance. Cowlin [1] lists misalignment as one of the principal cause for deviation of bearing performance from that predicted by hydrodynamic theory. Pigott [2] has observed that a misalignment or deflection of as small as 0.002 inch in 12 inch length can reduce the safe load carrying capacity by 40%. Buske and rolli [3] have observed the variation in film pressure in a bearing is the most sensitive indication of misalignment. In a test bearing 1.75 inch long they could produce a non symmetrical pressure pattern by tilting one edge of the bearing by only 0.00025 inch. Some experimental data have been reported by Du Bois [4] against skew represented by percentage of axial and twisting misalignment couples.

Buske [5] shows that the hydrodynamic pressure pattern in a bearing is very sensitive to misalignment. Khrisanova [6] has presented an experimental and analytical investigation of the influence of skewed axis of journal and bearing on the load carrying capacity. He has treated the problem analytically by the finite difference method.

Capiz [7] has analyzed the Reynolds equation for short misaligned journal bearing in terms of Euler angles for dynamic loading. D. V. Singh et.al [8] presented more general form of solution of Reynolds equation for misaligned finite journal bearing with static loading using Galerkin's method which is not restricted to an 180° extent of film. Kikuchi [9] has analyzed the unbalanced vibrations of rotating shaft system with many bearings and discs with the help of transfer matrix method. This analysis is for misaligned short circular journal bearing.

II. Objective of the Work
The objective of this work is to analyze the static and dynamic performance of misaligned finite circular journal bearing. The Reynolds equation for misaligned finite circular journal bearing is solved with the help of Galerkin's methods and the numerical values of pressure distribution in circumferential and axial direction, load carrying capacity and attitude angle are plotted.

III. Film Thickness
The approach of the analysis is taken from [8] and [9]. The coordinate system is set as shown in fig. 1. Equation of fluid film thickness for misaligned journal can be written as

$$H=1+\cos X'-\frac{\sigma Z}{C}\cos X'+\frac{\delta Z}{C}\sin X' \quad (1)$$

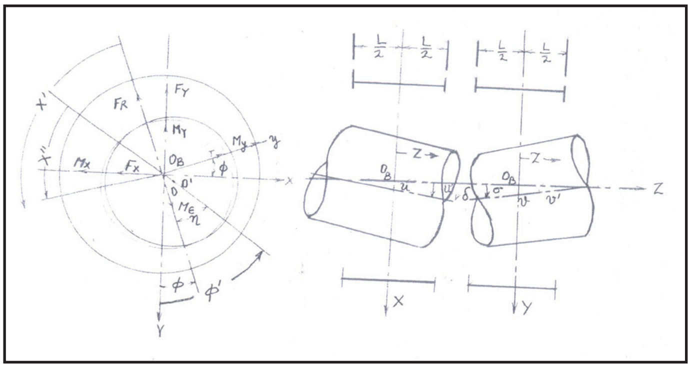

Fig. 1: Co-ordinate System

IV. Reynolds Equation
Reynolds equation for misaligned finite circular journal bearing in non-dimensional form for static and dynamic loading can be written as:

$$\frac{\partial}{\partial X'}\left(H^3\frac{\partial P}{\partial X'}\right)+\frac{\partial}{\partial Z}\left(H^3\frac{\partial P}{\partial Z}\right)=12\pi\,[2\dot{\epsilon}\cos X'-\epsilon\sin X'+\frac{Z}{C}(\delta\cos X'+\sigma\sin X')+2(\dot{\delta}\sin X'-\dot{\sigma}\cos X')] \quad (2)$$

V. Boundary Conditions
Since in X' direction, due to the presence of skew, the boundaries at the beginning and at the end of the positive pressure zone are assumed to deviate from the line X' = 0 and X' = θ respectively by an angle γ. The boundary conditions are:

$$P\,(X' = KZ,\ Z) = 0$$
$$P(X'=\theta+KZ, Z)=0 \quad (3)$$
$$P\,(X', \pm B) = 0$$

Where θ may have any value including π, representing the extent of positive film and
K= tan γ

VI. Solution of Reynolds Equation
Using the Galerkin's method a solution of Reynolds equation satisfying the boundary conditions can be established, in terms of the following form of linearly independent, complete, continuous, and orthogonal function

$$P=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}a_{nm}\sin\{\alpha_m\,(X'-KZ)\}\cos(\beta_n Z)$$

$$\text{Where}\quad \beta_n=\frac{(2n-1)\pi}{2B},\ \text{and}\quad \alpha_m=\frac{m\pi}{\theta} \quad (4)$$

Substituting the value of P from equation (4) in equation (2) we get

L(P)=$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{nm}[\gamma_{nm}^2$ H^{3}sin$\{\alpha_m\ (X' - KZ\ )\}\cos(\beta_n\ Z)$+2K$\alpha_m\beta_n$H^{3}sin $(\beta_n\ Z)$ cos $\{\alpha_m\ (X' - KZ\ )\}$ +3 H^2 $\frac{\partial H}{\partial X'}$ α_m cos$\{\alpha_m\ (X' - KZ)\}$ cos $(\beta_n\ Z)$ -3KH$^2\frac{\partial H}{\partial Z}$ α_m cos$\{\alpha_m\ (X' - KZ)\}\cos(\beta_n\ Z)$ − 3 β_n H^2 $\frac{\partial H}{\partial Z}$ sin $(\beta_n\ Z)$ sin $\{\alpha_m\ (X' - KZ\}]$ - 12π [2 $\dot{\overline{\epsilon}}$ cos X' − ϵ sin X' + $\frac{Z}{C}$ { (δ cos X' + σ sin X') + 2($\dot{\overline{\delta}}$ sin X' - $\dot{\overline{\sigma}}$ cos X')}] (5)

In accordance with the method of Galerkins

$\int_{-B}^{+B}\int_{KZ}^{\theta+KZ} L(P)\sin\{\alpha_i\ (X' - KZ\ )\}\cos(\beta_j\ Z)$dX' dZ =0 (6)

Substituting the value of L (P) from equation (5) in equation (6) we get

$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{nm}[\gamma_{nm}^2$ ∬ H^3 sin$\{\alpha_m\ (X' - KZ\ )\}\cos(\beta_n\ Z)$+2K$\alpha_m\beta_n$∫∫H^{3}sin $(\beta_n\ Z)$ cos $\{\alpha_m\ (X' - KZ\ )\}$ +3 α_m∬H^2 $\frac{\partial H}{\partial X'}$ cos$\{\alpha_m\ (X' - KZ)\}$ cos $(\beta_n\ Z)$ -3Kα_m ∬H^2 $\frac{\partial H}{\partial Z}$ cos$\{\alpha_m\ (X' - KZ)\}\cos(\beta_n\ Z)$ − 3β_n ∬H$^2\frac{\partial H}{\partial Z}$sin $(\beta_n\ Z)$sin $\{\alpha_m\ (X' - KZ\}]$

sin$\{\alpha_i\ (X' - KZ\ )\cos(\beta_j\ Z)$dX'dZ = 12π [∬ 2 $\dot{\overline{\epsilon}}$ cos X' − ϵ sin X' + $\frac{Z}{C}$ { (δ cos X' + σ sin X')+ 2($\dot{\overline{\delta}}$sinX'- $\dot{\overline{\sigma}}$cosX')}]sin$\{\alpha_i\ (X' - KZ\ )\cos(\beta_j\ Z)$dX'dZ (7)

Or$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{nm}[\gamma_{nm}^2\ I1$ +2K$\alpha_m\beta_n I2$+3$\alpha_m I3$ − 3$\beta_n\ I4$-3K$\alpha_m\ I5$ =12π$I6$ (8)

Or $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{nm}$ A_{nmij} = βij (9)

Where

A_{nmij} = $\gamma_{nm}^2\ I1$+2K$\alpha_m\beta_n I2$+3$\alpha_m I3$ -3K$\alpha_m I4$ −3$\beta_n I5$ (10)

βij =12π$I6$ (11)

Where $I1$= ∬H^{3}sin$\{\alpha_m\ (X' - KZ\ )\}\cos(\beta_n\ Z)$ sin$\{\alpha_i\ (X' - KZ\ )\}\cos(\beta_j\ Z)$dX'dZ (12)

$I2$= ∬H^{3}sin $(\beta_n\ Z)$cos $\{\alpha_m\ (X' - KZ\ )\}$ sin$\{\alpha_i\ (X' - KZ\ )\}\cos(\beta_j\ Z)$dX'dZ (13)

$I3$= ∬H$^2\frac{\partial H}{\partial X'}cos\{\alpha_m\ (X' - KZ)\}$cos $(\beta_n\ Z)$sin$\{\alpha_i\ (X' - KZ\ )\}\cos(\beta_j\ Z)$dX'dZ (14)

$I4$= ∬H^2 $\frac{\partial H}{\partial Z}cos\{\alpha_m\ (X' - KZ)\}$ $\cos(\beta_n\ Z)$ sin$\{\alpha_i\ (X' - KZ\ )\}$ $\cos(\beta_j\ Z)$ dX'dZ (15)

$I5$= ∬H$^2\frac{\partial H}{\partial Z}$sin $(\beta_n\ Z)$sin $\{\alpha_m\ (X' - KZ)\}$ sin$\{\alpha_i\ (X' - KZ\ )\}\cos(\beta_j\ Z)$dX'dZ (16)

$I6$=12π [∬2$\dot{\overline{\epsilon}}$cosX'− ϵsinX'+$\frac{Z}{C}$\{(δcosX'+σsinX')+2($\dot{\overline{\delta}}$sinX'-$\dot{\overline{\sigma}}$cosX')}] sin$\{\alpha_i\ (X' - KZ\ )\cos(\beta_j\ Z)$dX'dZ (17)

VII. Fluid Film Forces

The non dimensional fluid film forces along and perpendicular to the line of centers of the bearing and journal are F_R and F_T respectively, are given by

F_R=$-\int_{-B}^{B}\int_{KZ}^{\theta+KZ} P\cos X' dX' dZ$ (18)

Or F_R = $-\int_{-B}^{B}\int_{KZ}^{\theta+KZ}[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{nm}\sin\{\alpha_m\ (X' - KZ\ )\}\cos(\beta_n\ Z)\cos X']dX'dZ$ (19)

Or F_R = $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{nm}\ I7$

Where,

$I7 = -\int_{-B}^{B}\int_{KZ}^{\theta+KZ}$ [sin$\{\alpha_m\ (X' - KZ)\}$ $\cos(\beta_n\ Z)\cos X']dX'dZ$ (20)

Similarly,

F_T =$\int_{-B}^{B}\int_{KZ}^{\theta+KZ}[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{nm}\sin\{\alpha_m\ (X' - KZ\ )\}$ $\cos(\beta_n\ Z)\sin X']dX'dZ$ (21)

OrF_T =$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_{nm}\ I8$ (22)

Where$I8 = \int_{-B}^{B}\int_{KZ}^{\theta+KZ}$ [sin$\{\alpha_m\ (X' - KZ)\}$ $\cos(\beta_n\ Z)\sin X']dX'dZ$ (23)

Load carrying capacity W = √(F_R^2+F_T^2) (24)

Attitude angle is given by tan φ = F_T/F_R (25)

VIII. Results and Discussion

The numerical values of non-dimensional pressure, load capacity, and attitude angle have been plotted for several values of eccentricity ratio for L/D = 1.0, σ = 23.75×10^{-5} , S= -52.09 ×10^{-5}, γ = -40° and C= 17.6×10^{-4}. Numerical value of constant was selected to compare the results with Singh et.al [8].

Using equation (4), the numerical valves of non- dimensional pressures are plotted as function of circumferential direction. It is found that pressure increases with X' and it becomes maximum for X'= 0.8 and reduces to zero for X' = 1. It is also found that for Z= -0.4 (towards left of journal center) the maximum pressure reduces and shifts towards right. For Z= 0.4(towards right of journal center) the maximum pressure reduces and shifts towards left. (fig. 2).

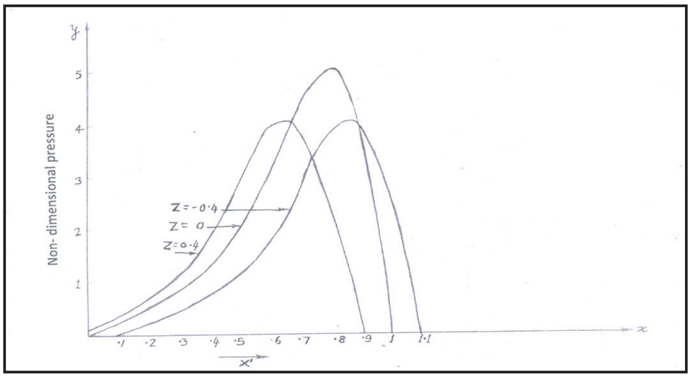

Fig. 2: Dimensional Pressure V/S Circumferential Direction

Fig. 2, Non Using equation (24), load capacity W is plotted. It is found that W increase with ϵ and it becomes 50 at ϵ= 0.9. (Fig. 3)

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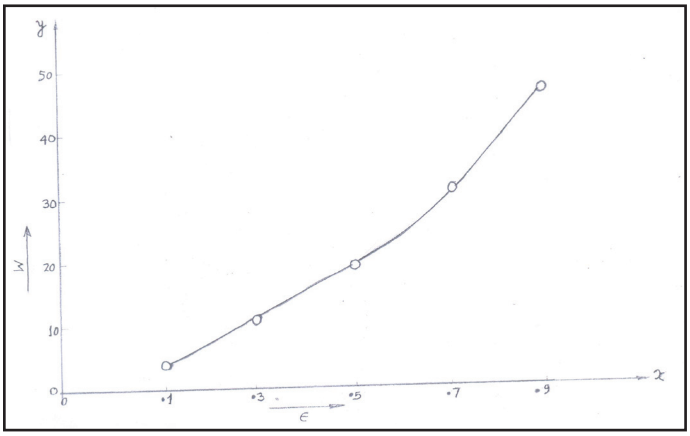

Fig. 3: Load Capacity V/S Eccentricity Ratio

Using equation (25), attitude angle is plotted with ϵ. It is found that attitude angle decreases with ϵ. In the beginning the attitude angle is around 90° and it becomes around 53° for ϵ = 0.52. (Fig. 4)

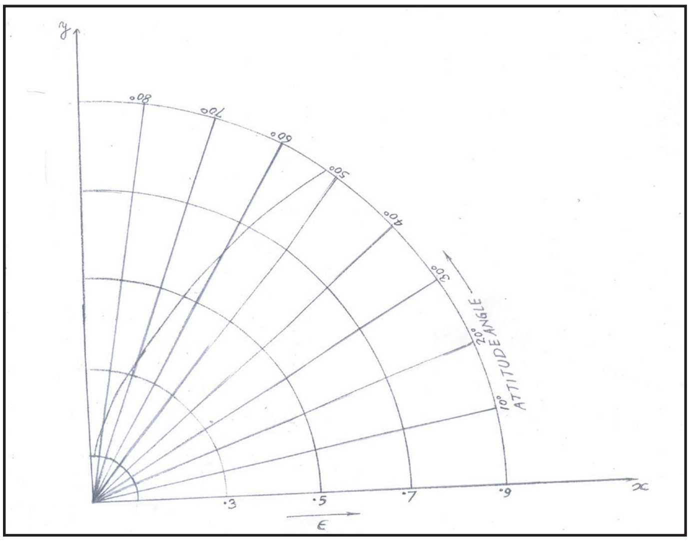

Fig. 4: Attitude Angle V/S Eccentricity Ratio

IX. Conclusion

The present work is for static as well as for dynamic loading. The work can be further extended for finding direct and rotational spring and damping coefficients. With the help of these coefficients stability of journal can be analyzed.

Nomenclature

A_{nmij}	Constants for Various m, n, i, and j
a_{nm}	Coefficients of series solution
B	L/D
B_{ij}	Constants for various i and j
c	Radial clearance
C	c/R
D	Journal diameter
e	Eccentricity
F_R	Radial force of fluid film
F_T	Tangential force of fluid film
F_X	Horizontal component of the force
F_Y	Vertical component of the force
h	Film thickness
H	h/c
i , j	Dummy indices for n and m respectively
K	tan (γ)
L(P)	Error function
m, n	Indices
O_B	Bearing center
O	Journal center at middle section
O’	Center of journal section at distance Z from middle section
P	$2\pi p/[\mu(R/c)^2(\omega - 2(\beta)]$, Nondimentional pressure
p	Dimensional fluid film pressure
R	Journal radius
S	Summerfield number
t	Time
U	Journal velocity
u, v	Coordinate of O
u’, v’	Coordinates of O’
ω	Rotational velocity of journal
X’	x/R, Circumferential direction
x	Direction joining journal center and O
X	Horizontal direction
Y	Vertical direction
z	Axial direction
Z	z/R
φ'	Angle as shown in Fig. 1
φ	Attitude angle
ϵ	e/c
σ	Rotation about x- axis in counter clock wise direction
δ	Rotation about y- axis in counter clock wise direction
Θ	Extent of film
μ	Absolute viscosity
γ	Angle by which the boundary of pressure zone deviates from the line X= 0 and X= Θ at beginning and at the end respectively
γ^2_{nm}	$-\alpha_m^2 - (K^2\alpha_m^2 + \beta_n^2)$
α_m	$m\pi/\Theta$
α_i	$i\pi/\Theta$
β_n	$\frac{(2n-1)\pi}{2B}$
β_i	$\frac{(2i-1)\pi}{2B}$
$\bar{\dot{\epsilon}}$	$\dot{\epsilon}/(\omega - 2\dot{\varphi})$
$\bar{\dot{\delta}}$	$\dot{\delta}/(\omega - 2\dot{\varphi})$
$\bar{\dot{\sigma}}$	$\dot{\sigma}/(\omega - 2\dot{\varphi})$
$\dot{\epsilon}$	Radial velocity of journal center
$\dot{\varphi}$	Circumferential velocity of the journal

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