

# Repair of Notched Cantilever Beam by Piezoelectric Material

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## Abstract

Mechanical structures during their functional operations may be vulnerable to damage and therefore cannot be guaranteed definite fault free operational and successful exploitation. This paper presents the modal analysis for the use of PZT actuator in repair of cracked cantilever aluminium beam by using Finite Element Analysis (Ansys) software. Cantilever beam vibrational response is analysed and the numerical results of undamaged beam, damaged beam and damaged beam with piezoelectric material patch at different locations compared to different scenarios of damage presence in structure, by location and depth of single transversal notch. Technique is based on idea, if notch appears in mechanical structure which leads to change in physical properties leading to the drop in natural frequency. By using the piezoelectric patch the dropped natural frequency are tried to be restored to the Natural frequency of healthy beam.

## Keywords

Piezoelectric Material, Healthy Beam, Damaged Beam, Natural Frequency

## I. Introduction

Smart structures are a rapidly advancing field with the range of support and enabling technologies having significant advances, notable optics and electronics. The definition of smart structure was a topic of controversy from the late 1970 to 1980. In order to define this a special workshop was organized by the US army research office in 1988 in which Sensors, Actuators, Control mechanism and Timely response were recognized as the four qualifying features of any smart system or structures. In this workshop Smart structure is defined as "A system or material which has built in intrinsic Sensor, actuator and control mechanism whereby it is capable of sensing a stimulus, responding to it in a predetermined manner and extent, in a short time and reverting to its original state as soon as the stimulus is removed." According to Spilman a smart structure is defined as "a physical structure having a definite purpose, means of imperative to achieve that purpose and the pattern of functioning of a computer. "Smart structure contains a host structure, a sensor to gauge its internal state, an actuator to affect its internal state and, a controller whose purpose is to process the sensors and appropriately send signals to actuators. Vibration control is an important area of interest in several industrial applications. Unwanted vibration can have a detrimental and sometimes catastrophic effect on the serviceability or structural integrity of mechanical systems. To control the vibrations in a system, different techniques have been developed. Some of these techniques and methods use piezoelectric materials as sensors or actuators. A vibration isolation system is called active if it uses external power to perform its function. It consists a servomechanism with a sensor, actuator, and signal processor. Active control systems are required in applications where passive vibration control is not possible because of material constraints or simply not sufficient for the level of control required. Active control is a favourable method of control because it works in a wide frequency range, reducing resonant vibrations within that range and because it is adaptive to changes in the nature of

the disturbance. Active smart materials are those materials which possess the capacity to modify their geometric or material properties under the application of electric, thermal or magnetic field, thereby acquiring an inherent capacity to transducer energy. The active smart materials are piezoelectric material, Shape memory alloys, Electro-rheological fluids and Magneto-structive materials. Being active they can be used as force transducers and actuators. The materials which are not active under the application of electric, thermal or magnetic field are called Passive smart materials. Fibre optic material is good example of passive smart material. Such materials can act as sensors but not as actuators and transducers.

## A. Piezoelectric Material

Piezoelectricity is the ability of a material to develop an electric charge when subjected to a mechanical strain, this effect is called Direct Piezoelectric Effect (DPE) and Conversely material develop mechanical strain in response to an applied electric field, this effect is called Converse Piezoelectric Effect (CPE). Due to this coupled mechanical and electrical properties, piezoelectric materials make them well suited for use as sensors and actuators. Sensors use Direct Piezoelectric Effect (DPE) and actuators use Converse Piezoelectric Effect (CPE). As a sensors, deformations cause by the dynamic host structure produce an electric change resulting in an electric current in the sensing circuit. While as an actuators, a high voltage signal is applied to piezoelectric device which deforms the actuator and transmit mechanical energy to the host structure. Piezoelectric materials basically divided into two group Piezo-ceramics and piezo-polymers.

### 1. Piezo-Ceramics

The most common commercial piezo-polymer is Barium Titanate ( $\text{BaTiO}_3$ ), Lead Titanate ( $\text{PbTiO}_3$ ), Lead Zirconate ( $\text{PbZrO}_3$ ) Lead metaniobate ( $\text{PbNb}_2\text{O}_6$ ) and Lead (plumbum) Zirconate Titanate (PZT) [ $\text{Pb}(\text{ZrTi})\text{O}_3$ ]. Among these materials last Lead (plumbum) Zirconate Titanate (PZT) became the dominant piezo-electric ceramic material for transducer due to its high coupling coefficient (0.65). When this PZT plate subjected to static or dynamic loads, it can generate voltages as high as 20,000 volts.

### Examples

Microphones, headphones, loudspeakers, buzzers, wrist watches, clocks, calculators, hydrophones and projectors.

There have been many researches using finite element method to analyse piezoelectric structures. Allik and Hughes [1] applied the finite element method to analyse the three-dimensional piezoelectric vibration modes. The finite element formulation included the piezoelectric and electro elastic effect. A tetrahedral finite element was also presented based on three dimensional electro-elasticity. Boucher et al. [2] developed a perturbation method to numerically determine the Eigen modes of vibration for piezoelectric transducers. The three-dimensional finite element method was formulated to predict the piezoelectric transducer resonance and anti-resonance frequencies as well as sound radiation for different sizes of the PZT cubes. Kunkel et al. [3] applied the finite element method to calculate the natural vibration modes of

the piezoelectric ceramic disks. To optimize the disk geometry, the dependence of the vibration mode on the disk diameter-to-thickness ratio was studied. Ha et al. [4] modelled laminated composite structures containing distributed piezoelectric ceramic sensors and actuators by finite element analysis. The computer code was developed to analyse the mechanical-electrical response of the piezoelectric ceramic laminated composites. Experiments were also conducted to verify the computer simulations. The comparisons between predicted and experimental results agreed well. There were also many researches via Finite element analysis about piezoelectric ultrasonic transducers and piezoelectric transformers such as Kagawa and Yamabuchi [5], Challande [6] and Tsuchiya and Kagawa [7]. The adoption of piezoelectric transducers for structural modal testing is also drawn attention. Sun et al.[8] derived the frequency response function (FRF) through electric admittance of piezoelectric transducers for obtaining the dynamic parameters of beam structures. However, they did not physically interpret those dynamic parameters. Norwood [9] successfully applied both the impact hammer and PVDF film as actuation sources, respectively, to modal testing of cylindrical shell structures. Wang [10] derived the frequency response functions between the traditional and piezoelectric transducers for simply supported beam. He introduced the feasibility of the use of piezoelectric transducers for structural modal testing. Wang [11] generalized the formulation of frequency response functions (FRFs) for continuous structure systems subject to various forms of actuators and sensors. The actuator and sensor Eigen functions (mode shapes) were respectively identified and physically interpreted according to the testing procedures, either roving the actuator or the sensor. Wang’s work provided with the theoretical base for the application of smart materials, such as PZT actuators and PVDF sensors, to smart structural testing. Wang and Wang [12] theoretically demonstrated the feasibility of using piezoelectric transducers for cantilever beam modal testing. An array of finite-length PVDF films was assumed to be equally spaced and distributed over the beam acting as sensors, while a fixed pure-bending PZT actuator was served as actuation force. They performed synthetic modal analysis to extract modal parameters of the beam by using piezoelectric transducers. Wang [13] also applied the similar arrangement of an array of PVDF sensors on a cantilever beam and developed a novel wave number domain sensing technique for active structural acoustic control (ASAC). Many methods have been developed to detect and locate the crack by measuring the change in the natural frequencies of the structure due to modal frequencies are properties of the whole structure and decreases as a result of crack. Comprehensive survey for detection, location, and characterization of structural damage via techniques that examine changes in measured structural vibration response, are presented by S. W. Doebling et al. [14]. The survey categorizes the methods according to required measured data and analysis technique, changes in modal frequencies, mode shapes, and changes in measured flexibility coefficients. Methods that use property (stiffness, mass, damping) matrix updating, detection of nonlinear response, and damage detection via neural networks are also presented. The types of structures include in this survey are: beams, trusses, plates, shells, bridges, offshore platforms, other large civil structures, aerospace structures, and composite structures. In this paper, natural frequencies have been calculated using finite element method in commercially available software package ANSYS. Modal analysis and frequency response analysis in different cases of damage presence on cantilever aluminium beam with bonded piezoelectric transducer are presented.

## II. Finite Element Analysis

Cantilever beam model was created in software for finite element analysis ANSYS 14.5. The beam model is based from laboratory set-up experiment for cantilever aluminium beam with following dimensional properties in the Table 1:

Table 1: Properties of Beam

Thickness $b$	0.002 m
Height $h$	0.035 m
Length from fixed end $l$	0.88 m
Young’s modulus $E$	$69 \times 10^9$ N/m <sup>2</sup>
Density $\rho$	2700 kg/m <sup>3</sup>
Poisson ratio $\mu$	0.35.

Model of damage beam is created and damage is presented by single transversal notch, and it’s assumed always to be open during dynamic analysis. To find out how the notch affects the dynamic behaviour of the beam, different notch scenarios are obtained by two notch parameters, different depth  $d$  and at different locations  $l_c$  (different distance measured from the fixed end), shown in Fig. 1. Before using the piezoelectric patch for the repair of the cracked beam. The obtained notched beam results of this paper are checked with the Reference results obtained from Marjan Djidrov. [15] which are show in below from fig. [2.0-10.0]. From these graphs as the ratios of healthy beam natural frequencies ad notched beam natural frequencies are obtained in are satisfactory at different the notch depths at different notch locations. So the further work is carried out on the repair of this damaged beam by using piezoelectric material. Here  $d_1, d_2, d_3$  are notch depths and  $L_1, L_2, L_3$  are notch locations from fixed end.

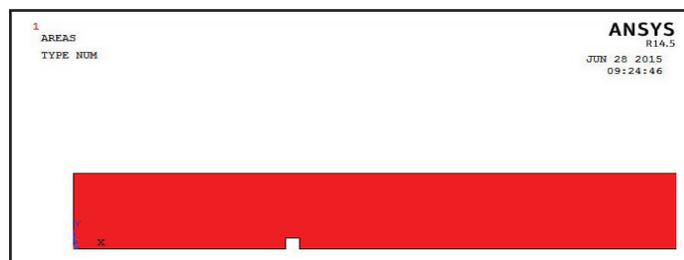


Fig. 1: Cantilever Beam with Notch

The graphs below shows the results comparison between reference result from Marjan Djidrov. [15] and this paper obtained results.

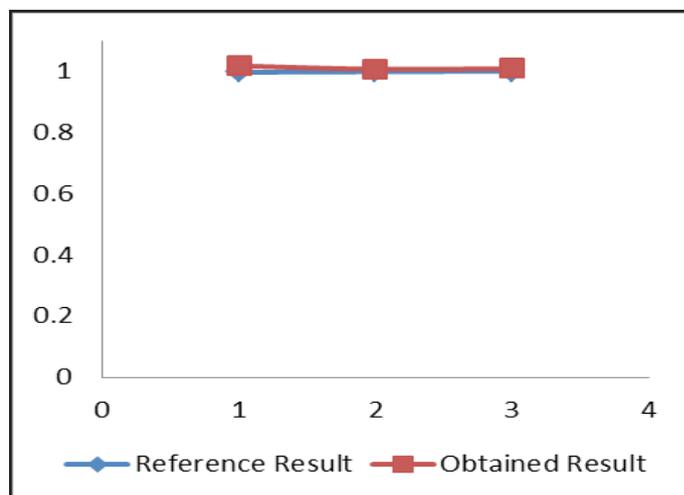


Fig. 2: Ratios of Natural Frequencies at  $L_1 = 0.079$  m &  $d_1 = 0.005$ m.

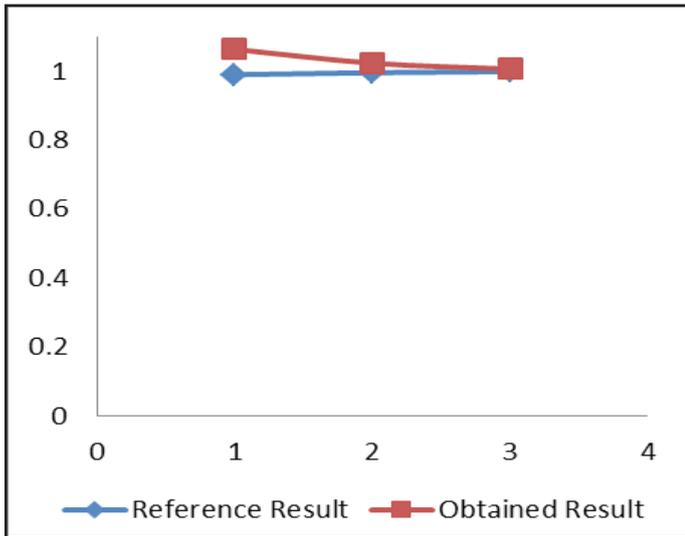


Fig. 3: Ratios of Natural Frequencies at  $L_1=0.079m$  &  $d_2=0.01m$ .

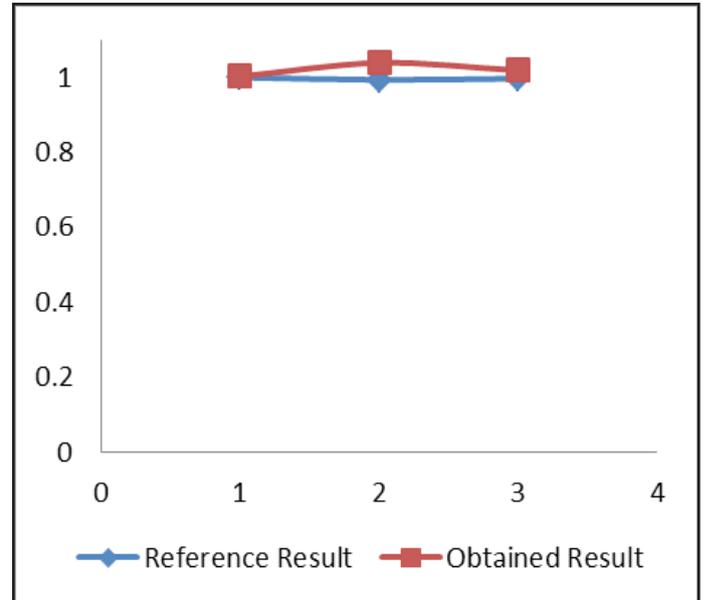


Fig. 6: Ratios of Natural Frequencies at  $L_2=0.52m$  &  $d_2=0.01m$ .

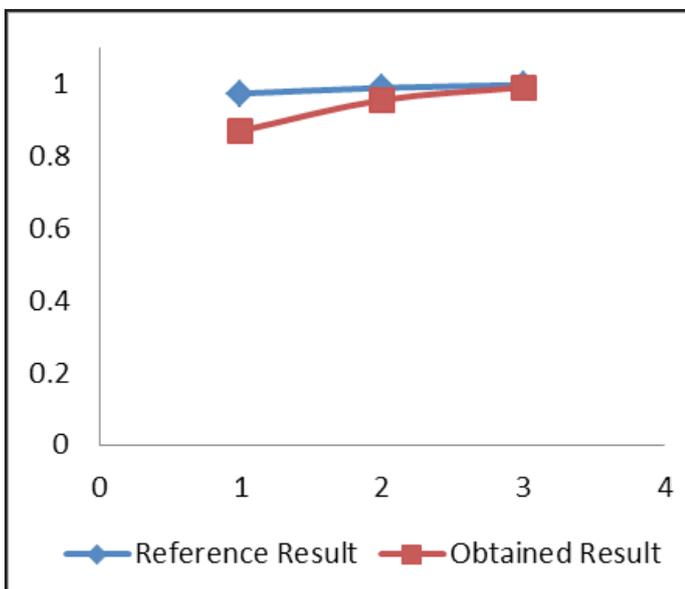


Fig. 3: Ratios of Natural Frequencies at  $L_1=0.079m$  &  $d_3=0.015m$ .

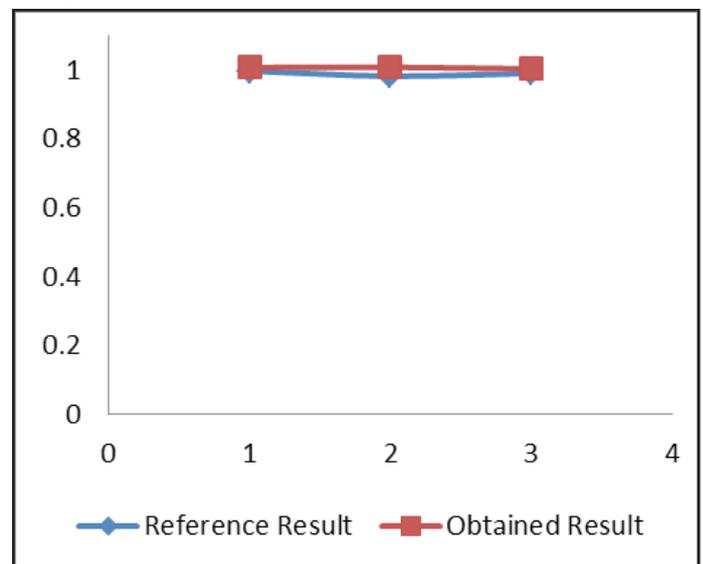


Fig. 7: Ratios of Natural Frequencies at  $L_2=0.52m$  &  $d_3=0.015m$ .

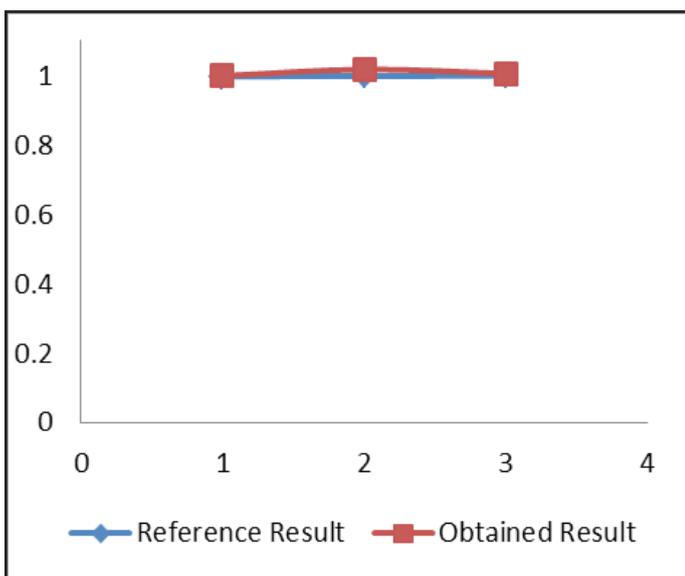


Fig. 5: Ratios of Natural Frequencies at  $L_2=0.52m$  &  $d_1=0.005m$

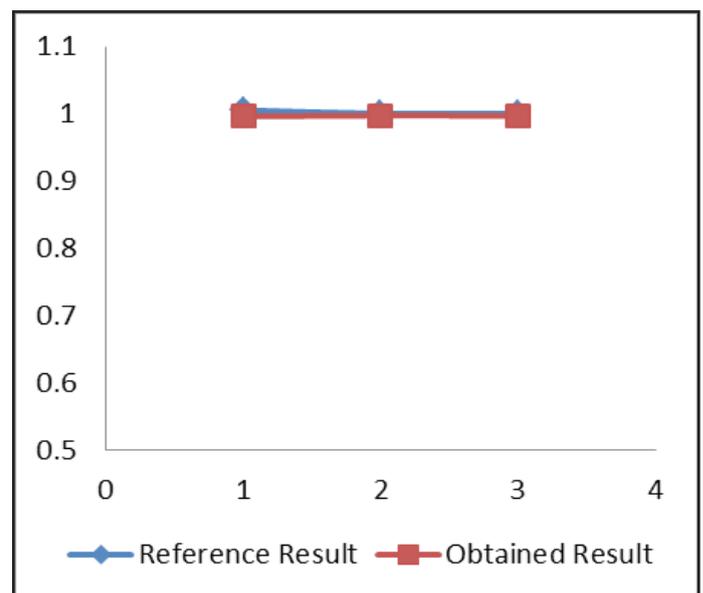


Fig. 8: Ratios of Natural Frequencies at  $L_3=0.86m$  &  $d_1=0.005m$ .

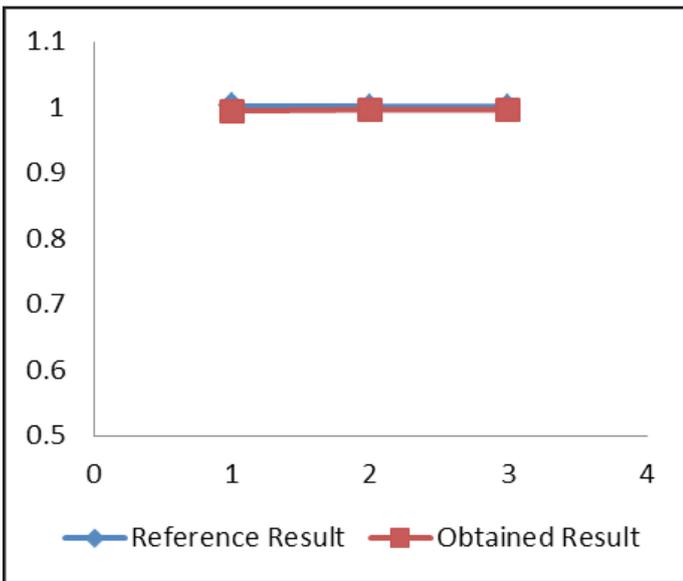


Fig. 9: Ratios of Natural Frequencies at  $L_3 = 0.86\text{m}$  &  $d_2 = 0.01\text{m}$ .

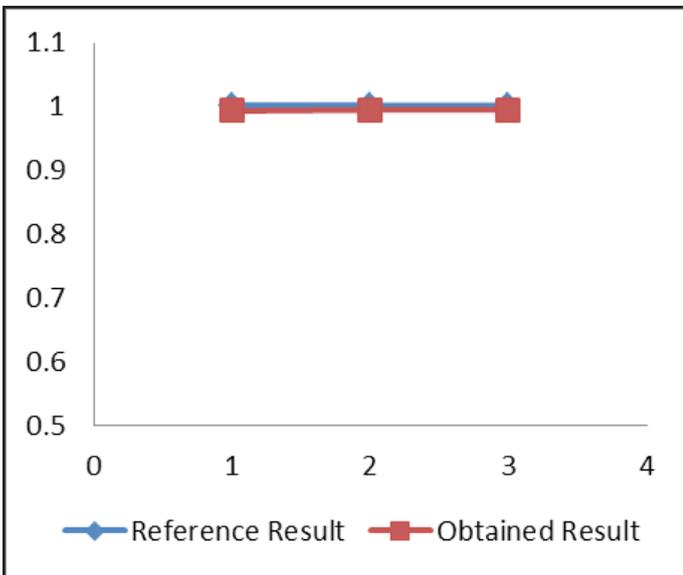


Fig. 10: Ratios of Natural Frequencies at  $L_3 = 0.86\text{ m}$  &  $d_3 = 0.015\text{m}$ .

**III. Simulation Procedure**

Measured natural frequencies are used for detection process only and through further analysis the same natural frequencies can be used for identification of notch location and size. Determination of the natural frequencies at higher modes is often difficult and only the first four natural frequencies were obtained by simulation of the un-notched, for all cases of notched cantilever beam models for the notch depth varied from 5 mm to 15 mm at each notch position of  $L_1=0.079\text{m}$ ,  $L_2=0.52\text{m}$ ,  $L_3=0.86\text{m}$  and in all the cases of the cantilever beam model where the notch depth varied from 5 mm to 15 mm, a piezoelectric patch is placed at each notch with lengths from the fixed end as shown in fig. 11.

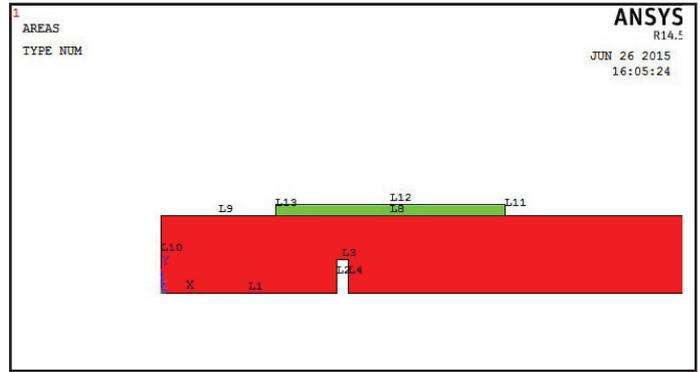


Fig. 11: Notched Beam with Piezo-electric Patch

The aluminium beam is modelled by using Plane183. The piezoelectric patch is modelled by using plane 223.

The piezoelectric material (PZT) of length=0.1m, Breath=0.005m, Thickness=0.002m Properties are mentioned in tables [2.0-4.0]

Table 2: Anisotropic Properties

Linear Elastic Anisotropic properties(N/m <sup>2</sup> )	
$D_{11}$	$1.26 \cdot 10^{11}$
$D_{12}$	$8.41 \cdot 10^{10}$
$D_{13}$	$7.95 \cdot 10^{10}$
$D_{22}$	$1.17 \cdot 10^{11}$
$D_{23}$	$8.41 \cdot 10^{10}$
$D_{33}$	$1.2 \cdot 10^{11}$
$D_{44}$	$2.3 \cdot 10^{10}$
$D_{55}$	$2.3 \cdot 10^{10}$
$D_{66}$	$2.35 \cdot 10^{10}$

Table 3: Electromagnetic Propertie

Electromagnetic Relative Permittivity(F/m)	
$\epsilon_{11}$	$1.151 \cdot 10^{-3}$
$\epsilon_{22}$	$1.043 \cdot 10^{-3}$
$\epsilon_{33}$	$1.151 \cdot 10^{-3}$

Table 4: Piezoelectric Properties

Piezoelectric constant stress matrix(C/m <sup>2</sup> )	
$e_{12}$	-5.4
$e_{22}$	15.8
$e_{32}$	-5.4
$e_{41}$	12.3
$e_{53}$	12.3

## A. Numerical Results

The variation of the frequencies as a function of the notch depth and notch location for cantilever beam models is plotted at different piezoelectric patch location for different notch depth at different location from tables [5.0-13.0].

At  $L_1=0.079\text{m}$  is patch is placed at  $l_1=0.079\text{m}$  &  $l_2=0.78\text{m}$ .

Table 5: Natural Frequencies at  $d_1=0.005\text{m}$

Mode	$f_{hb}$	$f_{cb}$	$f_{pl1}$	$f_{pl2}$
1	36.833	36.206	38.702	33.642
2	229.42	227.86	231.06	217.92
3	634.89	634.35	629.77	615.71
4	1223.8	1226.1	1202.9	1197.8

Table 6: Natural Frequencies at  $d_2=0.01\text{m}$ .

Mode	$f_{hb}$	$f_{cb}$	$f_{pl1}$	$f_{pl2}$
1	36.833	34.654	36.924	32.175
2	229.42	224.25	228.05	214.11
3	634.89	631.3	627.79	612.82
4	1223.8	1223.8	1202.8	1197.7

Table 7: Natural Frequencies at  $d_3=0.015\text{m}$ .

Mode	$f_{hb}$	$f_{cb}$	$f_{pl1}$	$f_{pl2}$
1	36.833	32.007	34.943	29.588
2	229.42	218.9	223.31	208.2
3	634.89	628.21	624.55	608.31
4	1223.8	1225.5	1201.8	1194.7

$f_{hb}$  - Frequency healthy beam

$f_{cb}$  - Frequency cracked beam

$f_{pl1}$  - Frequency repaired beam with piezoelectric patch at  $l_1=0.079\text{m}$  &  $f_{pl2}$  - Frequency repaired beam with piezoelectric patch at  $l_1=0.78\text{m}$ .

At  $L_2=0.52\text{m}$  is patch is placed at  $l_1=0.079\text{m}$  &  $l_2=0.28\text{m}$ .

Table 8: Natural Frequencies at  $d_1=0.005\text{m}$ .

Mode	$f_{hb}$	$f_{cb}$	$f_{pl1}$	$f_{pl2}$
1	36.833	36.865	36.064	37.231
2	229.42	226.76	228.14	222.68
3	634.89	631.46	631.65	624.4
4	1223.8	1220.8	1214.9	1213.8

Table 9: Natural Frequencies at  $d_2=0.01\text{m}$ .

Mode	$f_{hb}$	$f_{cb}$	$f_{pl1}$	$f_{pl2}$
1	36.833	36.759	35.983	36.968
2	229.42	220.99	222.97	218.55
3	634.89	623.5	624.09	619.99
4	1223.8	1213.3	1207.7	1206.43

Table 10: Natural Frequencies at  $d_3=0.015\text{m}$ .

Mode	$f_{hb}$	$f_{cb}$	$f_{pl1}$	$f_{pl2}$
1	36.833	36.543	35.813	37.225
2	229.42	211.25	213.62	205.89
3	634.89	610.73	611.38	607.66
4	1223.8	1201.3	1193.8	1191.25

$f_{pl1}$  - Frequency repaired beam with piezoelectric patch at  $l_1=0.52\text{m}$  &  $f_{pl2}$  - Frequency repaired beam with piezoelectric patch at  $l_1=0.28\text{m}$ .

At  $L_3=0.86\text{m}$  is patch is placed at  $l_1=0.78\text{m}$  &  $l_2=0.5\text{m}$ .

Table 11: Natural Frequencies at  $d_1=0.005\text{m}$

Mode	$f_{hb}$	$f_{cb}$	$f_{pl1}$	$f_{pl2}$
1	36.833	36.865	36.064	37.231
2	229.42	226.76	228.14	222.68
3	634.89	631.46	631.65	624.4
4	1223.8	1220.8	1214.9	1213.8

Table 12: Natural Frequencies at  $d_2=0.01\text{m}$ .

Mode	$f_{hb}$	$f_{cb}$	$f_{pl1}$	$f_{pl2}$
1	36.833	36.759	35.983	36.968
2	229.42	220.99	222.97	218.55
3	634.89	623.5	624.09	619.99
4	1223.8	1213.3	1207.7	1206.43

Table 13: Natural Frequencies at  $d_3=0.015\text{m}$

Mode	$f_{hb}$	$f_{cb}$	$f_{pl1}$	$f_{pl2}$
1	36.833	36.543	35.813	37.225
2	229.42	211.25	213.62	205.89
3	634.89	610.73	611.38	607.66
4	1223.8	1201.3	1193.8	1191.25

$f_{pl1}$  - Frequency repaired beam with piezoelectric patch at  $l_1=0.78\text{m}$  &  $f_{pl2}$  - Frequency repaired beam with piezoelectric patch at  $l_1=0.50\text{m}$ .

## V. Conclusion

In this paper the repair of damaged cantilever beam model has been presented. Damage is introduced as a single transversal crack, at different crack depth and at different crack position. The repair is done by introducing a piezo electric patch at different positions for each and every crack at each and every depth and each every location. Vibration behaviour of the beam simulated in FEA software ANSYS and obtained results from the analysis shows that the frequencies that got decreased due to crack can be increased by use the piezoelectric patch. The Numerical results show that the natural frequencies at the first two modes in all the cases when the patch is at the crack location yield the repair of beam in an effective manner than the patch at other location.

For future work, using the right measurement conditions using of experimental tests and Mat-lab (MEMS) the further scope of increasing accuracy of results and the possibility of considering third and fourth modes for the repair can be inspected.

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