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Structural Size Optimization of Ten Bar Truss Using Finite Element Analysis For Minimum Weight and Maximum Stiffness

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Abstract

The main purpose of this paper is to optimize and analyze a 10 bar aluminum 6063-T5 truss. Existing structural aluminum truss was initially optimized for minimum weight and maximum stiffness, in constrained with allowable stresses, deflections and design variables as radius of the truss members with structural optimization software package. The existing geometry and loading conditions of the truss are sized to mimic a real world environment. The structural aluminum truss was optimized using the design optimization tool and design of experiments in ANSYS Workbench and it is extended to compare for best suitable truss geometry for minimum weight and maximum stiffness. Results obtained are discussed for margin of safety by considering with design constraints and design variables. Finally, it is concluded that optimized truss is with 81% reduction in weight 22% increase in maximum stiffness to weight ratio with existing truss showing there within the designed bounds.

Keywords

Truss, Size optimization, Design of experiments, ANSYS, Margin of safety, Objective function, Constraints & Variables.

I. Introduction

Optimization of truss structures is a popular topic in mechanical, civil, and structural engineering due to the complexity of problems and benefits to industry. Weight optimization of trusses is so important due to economic and sustainability considerations. Geometry, topology and sizing optimization is extensively found in literature [1]. Applications found in literature uses the traditional design variables containing node coordinates, elements connectivity and member cross sections.

II. Structural Optimization Problem

The aim of any structural optimization problem can be defined in the following way: to determine the best design for a given problem subject to certain restrictions. The design variables can be defined starting from the geometry, the topology or material properties of the structures. A set of derived parameters related to mechanical behaviour can also be obtained: strains, stresses, deflections, natural frequencies and loads... The cost (or objective) function is given by the proper choice of the design criterion. This function can be either minimized or maximized. From a mathematical point of view, this problem can be formulated in the following way:

$$\begin{cases} \min : F(X) \ \ with \ \ X = (x_1, x_2, \cdots, x_n) \\ subejct\ to: \ g_i(X) \le 0 \ with \ i = 1, \cdots, p \end{cases}$$

A. Application: Size Optimization of Truss Structures

Size optimization of truss structures deals with the determination of optimum values for member cross-section areas Ai that minimize the structural weight W. Thus, this size optimization problem with stress constraints may be formulated as follows:

$$\begin{cases} \min W(\mathbf{A}) = \sum_{i=1}^{i=n} \rho_i \ A_i \ l_i \text{ with } \mathbf{A} = (A_1, \cdots, A_n) \\ \text{subject to} \begin{cases} A_{min} \le A_i \le A_{max} & \text{for } i = 1, \cdots, n \\ \sigma_{min} \le \sigma_i \le \sigma_{max} & \text{for } i = 1, \cdots, n \\ u_{min} \le u_j \le u_{max} & \text{for } j = 1, \cdots, k \end{cases} \end{cases}$$

where n is the number of elements of the structure; k is the number of nodes; l_i is the length of the ith element; r_i is density, and A_i is cross-section area of the ith element. S_i is the axial stress in the ith bar and u_j is the nodal vertical displacement of node j.

III. Literature

The object of the 10-bar truss benchmark is to compare results to a simple, well defined structure with few variables and constraints [2]. While Pyrz (2004) presents this problem in the literature, it was not chosen for results comparisons because discrete variables were used for design variables. The comparative work (Romero, 2004) presents all necessary information to reconstruct the problem and the structure was analyzed using linear elastic techniques [3]. This structure is optimized for minimization of mass with a single loading case and stress constraints applied to every member. Displacement constraints also considered for this problem. A single material is used that exhibits the properties [4].

IV. Modelling

The dimensions, node numbers, element numbers, and loaded nodes for this structure are shown in fig. 1 & 2. The single load case values and material properties are shown. Ten design variables are considered for optimization (each element diameter) with a 200 mm to 1 mm. Each member is subjected to stress limitations of 172.4 MPa and vertical displacement of all nodes is limited to 50.8mm.

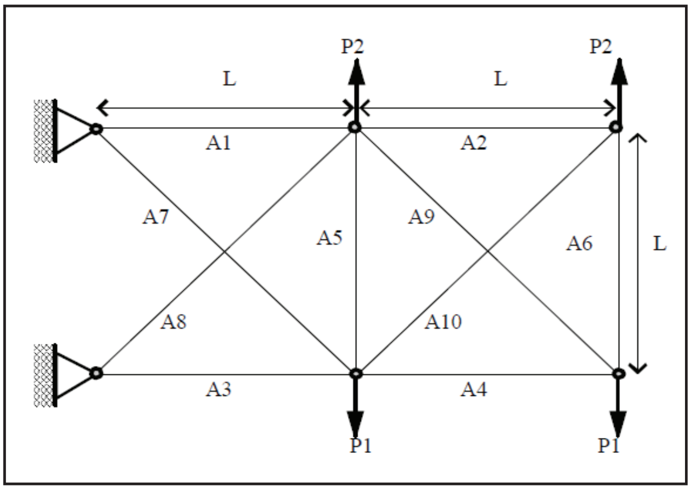

Fig. 1: Geometric Model of 10-bar Truss

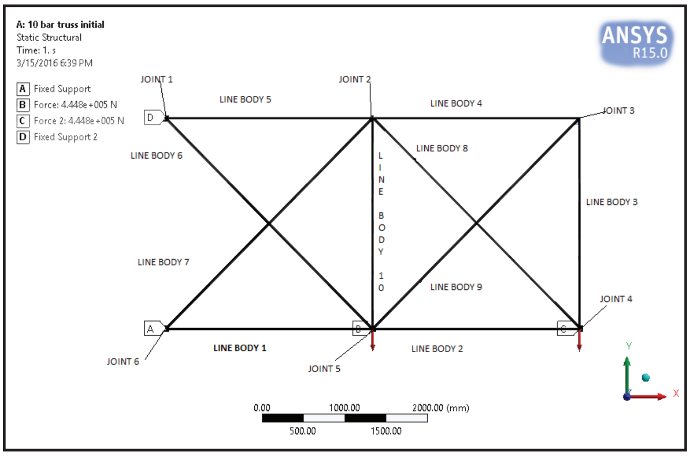

Fig. 2: Finite Element Model of of 10-bar Truss

Material Used [1]: Aluminum 6063-T5
E = 68.9 GPa, ρ = 2.8 e – 6 Kg/mm³
Length of each bar L= 2500 mm
P_1 = 444800 N, P_2 = 0 N

V. Results and Discussions

A. Pre-Optimization

A ten bar truss is modelled as ten line bodies in ansys work bench with six joints and loading conditions at two joints fully constrained at two joints as shown in fig. 3.

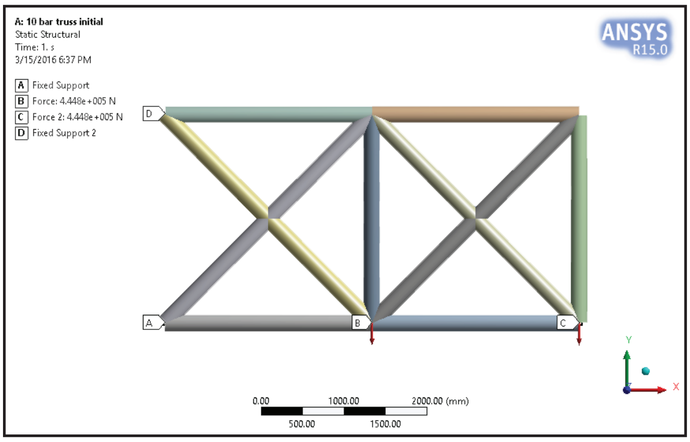

Fig. 3: Real Cross Section Area of the Truss

B. Nodal Displacements and stresses

After application of loads at two joints the following nodal displacements and stress in members are occurred. Here joints 3 & 4 are with maximum displacement and 1 & 5 line body members are with maximum stresses in which 5th member is in tension where as 1st member is in compression as shown in fig. 4 & 5

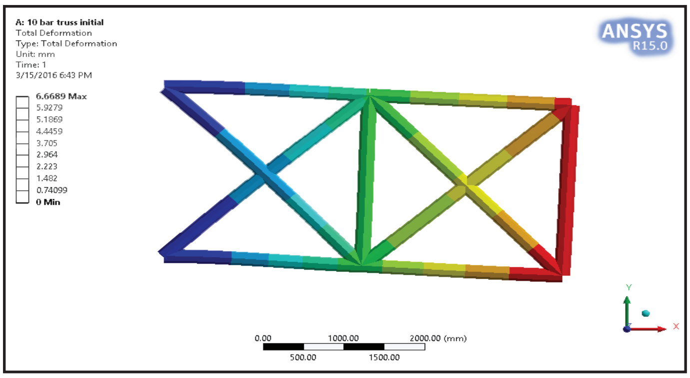

Fig. 4: Deformed Shape and Deflections

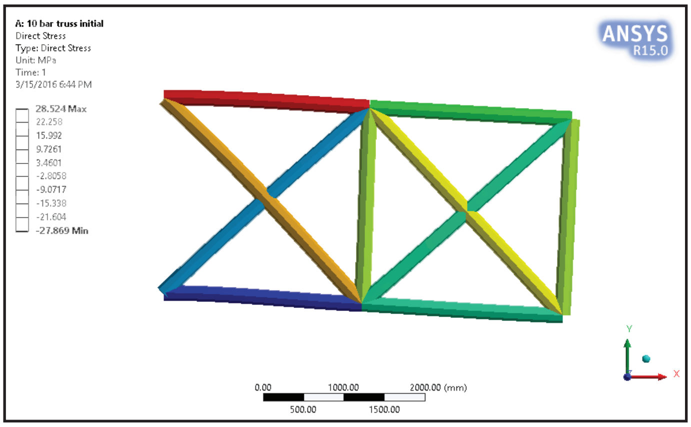

Fig. 5: Direct Stresses in Ten Bar Truss

C. Post-Optimization

Ten bar truss is optimized for same real time loading in Ansys work bench using optimization tool and design of experiments. over 14 iterations. Optimized model is shown in fig. 6.

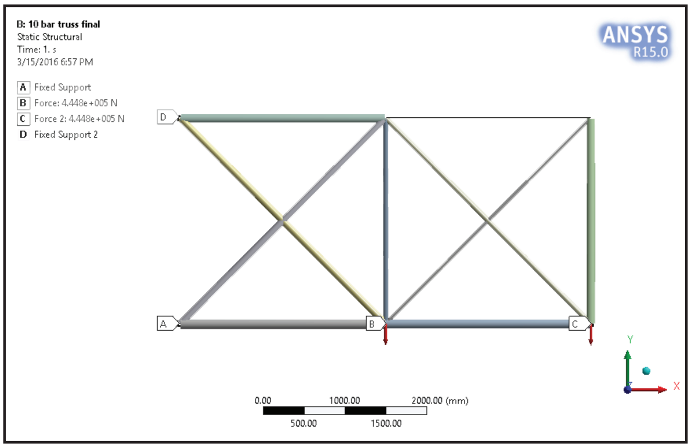

Fig. 6: Ten Bar Truss After Optimization

D. Nodal Displacements and Stresses

After optimization nodal displacements and stress in members are occurred. Here joints 3 & 4 are with maximum displacement and 6 & 9 line body members are with maximum stresses in which 6th member is in tension where as 9th member is in compression as shown in Fig. 7&8.

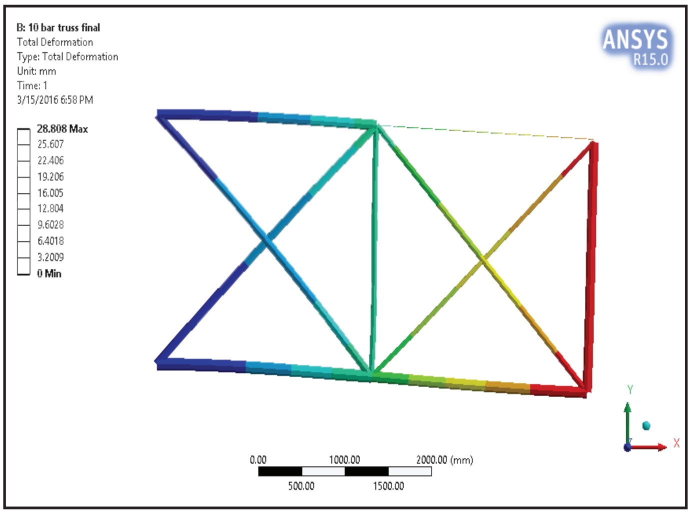

Fig. 7: Deformed Shape and Deflections

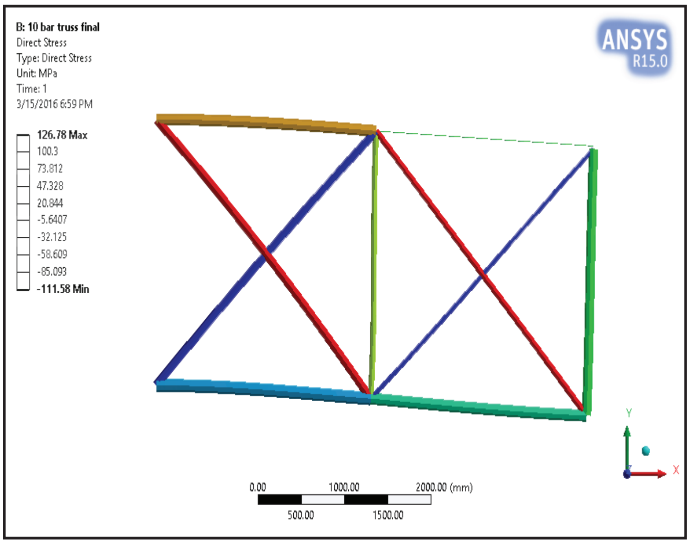

Fig. 8: Direct stresses in ten Bar Truss

E. Reduction in Cross-section area of an each member

The cross-sectional area of the each and every member is reduced after optimization leads to heavy reduction in mass. The comparisons are shown in fig. 9. It is also identified that cross sectional are of the member 4 tends to zero means zero force member.

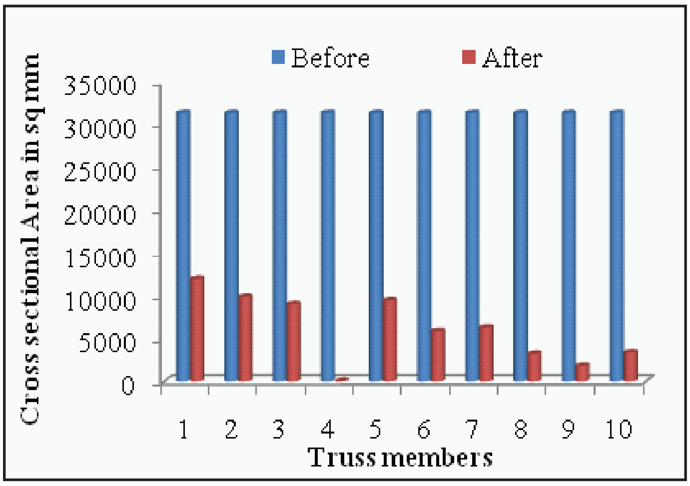

Fig. 9: Reduction in Cross-Sectional Areas

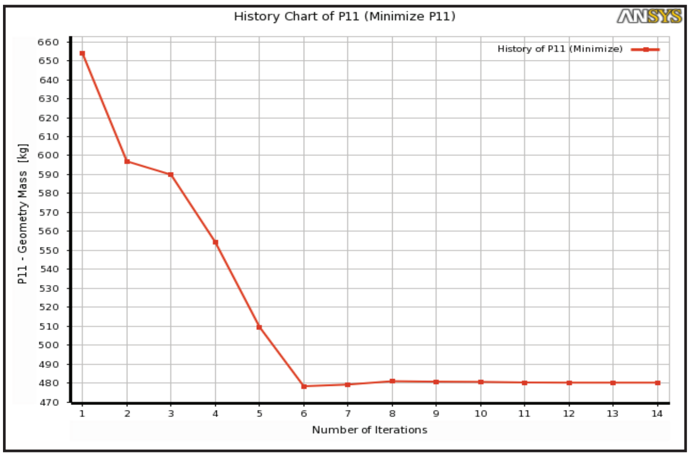

Fig. 10: Reduction in mass over 14 iterations

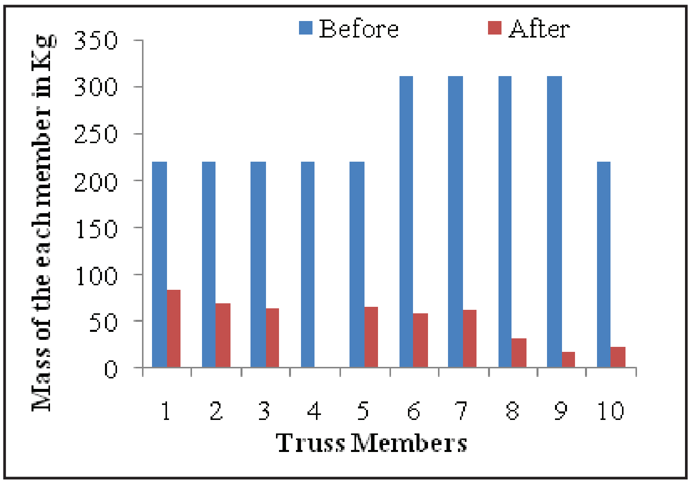

Fig. 11: Mass of Each Truss Members

F. Margin of Safety

The ultimate goal of using optimization for the construction of a truss is achieving a lower weight structure that has higher strength. Strength can be quantified by calculating a margin of safety in each truss member. Margin of safety is defined as in equation

$$M.S. = \frac{Allowable\ Strength}{Actual\ Strength} - 1$$

When the M.S. reaches a value less than or equal to zero the structure will fail. An alternate definition is that it is equal to the factor of safety. This factor is used to compare different trusses members. This is done by computing the M.S in every member of every truss model for axial and transverse direction. Then a comparison between the lowest axial M.S. in each truss member is made. The truss with the highest M.S. is deemed the strongest and the one with the lowest is ranked as the weakest. This results in a simple quantitative parameter to compare between each truss member.

G. Truss Margin of Safety

The material aluminium alloy 6063–T5 is an isotropic material. This means it has the same strength in every direction. The material allowable for this strength only varies with the type of stress applied. Table 1 gives details of the material allowable for compression and tension.

Table 1: Allowable Stresses and Deformations

Material	Allowable Stress (MPa)	Allowable Deformation
Aluminium	172.4	50.8 mm
Aluminium	-172.4	-50.8 mm

H. Design Constraint as maximum stress and its margin of Safety

The maximum axial stress in each member as computed was compared to the appropriate tension or compression material allowable. The member stresses used in this calculation can found in the previous section. The resulting axial M.S. is shown in Table 2. In truss members line body or truss member 3 is the strongest among all whereas greater care will be taken for line body or truss member 8 as defined weakest in Finite Element Analysis.

Table 2: Stress Margin of Safety Calculations

S.No	Truss	Pre-Optimum			Post-Optimum		
		Stress (MPa)	M.S	FOS	Stress (MPa)	M.S	FOS
1	1	-27.869	7.1	6.1	-75.558	3.3	2.3
2	2	-7.8038	22.8	21.8	-29.852	6.7	5.7
3	3	6.3866	25.6	26.6	16.587	9.3	10
4	4	0	-	-	0	-	-
5	5	28.524	5.0	6.0	91.588	0.9	1.9
6	6	19.299	7.8	8.8	110.01	0.5	1.5
7	7	-20.169	9.4	8.4	-94.738	2.8	1.8
8	8	10.943	14.6	15.6	126.78	0.3	1.3
9	9	-8.9079	20.1	19.1	-111.58	2.5	1.5
10	10	6.5476	25.0	26.0	38.099	3.5	4.5

I. Design Constraint as Nodal Displacements and its Margin of Safety

Truss nodal deflections given information about the truss deformations during loading. As more the deformations the loads are unstable during their loading. So, it is necessary the nodal deflections are as low as possible. As defined at joints 3 and 4 nodal displacements are high but with in the design constraint level. The resulting nodal displacements are shown in Table 3.

Table 3: Nodal Displacement Calculations

Node Joint	Pre -Optimum				Post-optimum			
	Ux	MS	Uy	MS	Ux	MS	Uy	MS
1	0	-	0	-	0	-	0	-
2	1.0	48	-2.8	19	3.5	13	-10.8	6
3	1.2	42	-6.0	9	4.1	11	-27.5	3
4	-1.5	34	-6.5	9	-4.3	13	-28.5	3
5	-1.1	47	-3.	17	2.9	16	-12.3	5
6	0	-	0	-	0	-	0	-

J. Design variable as radius of members and its margin of safety: Radius of truss members are optimized under given design variable ranges from 200 mm to 1 mm over 14 iteration shown in the previous sections. Line body or truss member 1 is showing high radius whereas Line body or truss member 4 is with low radius. The resulting radius and area of cross-sections are shown in Table 4

Table 4: Radius of the Circular Truss Members

Truss	Design variable radius bound ranges from 1 to 100 mm			
	Radius	Allowed	Cross Section	Bound Margin Ratio
1	61.818	100	12005	1.62
2	56.118	100	9893	1.78
3	53.729	100	9069	1.86
4	3.8327	100	46	26.09
5	54.984	100	9498	1.82
6	43.483	100	5940	2.30
7	44.923	100	6340	2.23
8	32.313	100	3280	3.09
9	24.543	100	1892.3	4.07
10	33.004	100	3422	3.03

K. Weight or material saving as objective function

Weight as an objective function is reduced excellently by 81% for similar loading conditions with reduction factor 5.34.

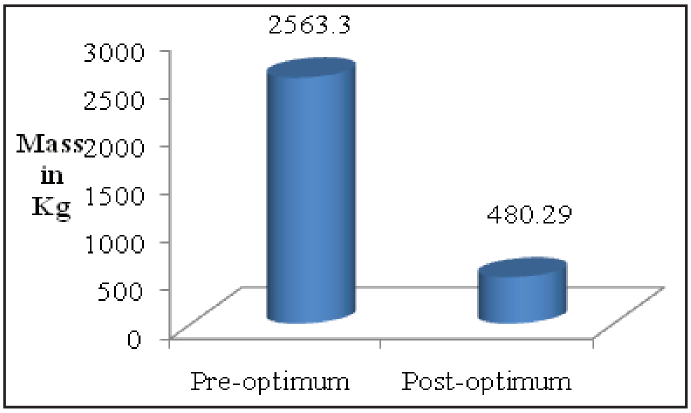

Fig. 9: Mass Reduced in Kilograms

L. Design Constraint as Deflection or Deformation

A deflection of the joints after optimization is 28.808 mm in which design constraint value as 50.8 mm with safety 1.76

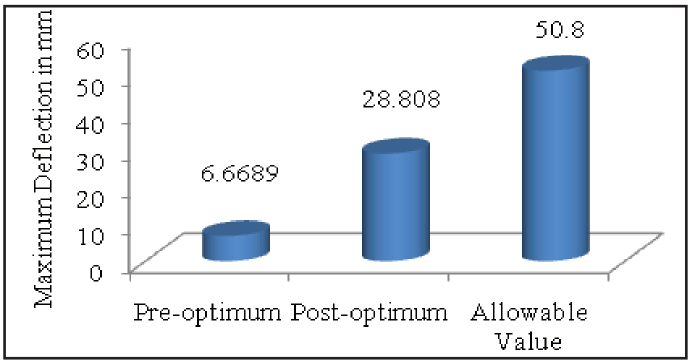

Fig 10: Maximum Deflection Over Constrained

M. Comparison of Maximum Deflection for Design Constraint:

Stiffness of the 10 bar truss is calculated for pre and post optimization with real load and tabulated in Table 5

Table 5: Pre and Post Deformations at Joints

Joints	Force applied KN	Pre deformation (mm)	Stiffness N/m	Post deformation (mm)	Stiffness N/m
2	0	-2.83	0	-10.86	0
3	0	-6.03	0	-27.59	0
4	444.8	-6.50	68.34	-28.52	15.6
5	444.8	-3.20	138.6	-12.38	36

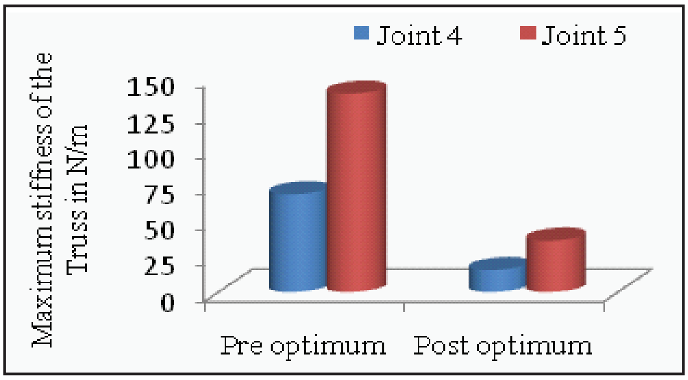

Fig 11. Reduced stiffness after optimization

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N. Design constraint as direct stress in truss members

Maximum stress in members are 126.78 MPa in which design constraint value as 172.4 MPa with safety 1.36

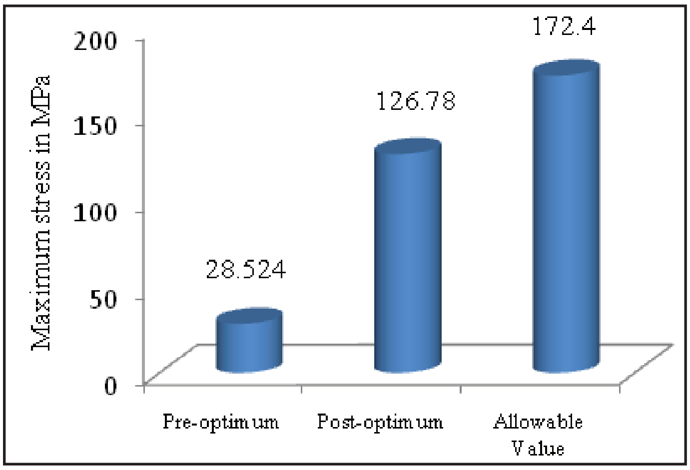

Fig. 12: Direct Stresses Over Constrained

O. Design Variable as Radius of the Truss Members

Radius as a design variable with bounds 1mm to 100mm leads to different radiuses for different members showing that within the bounds.

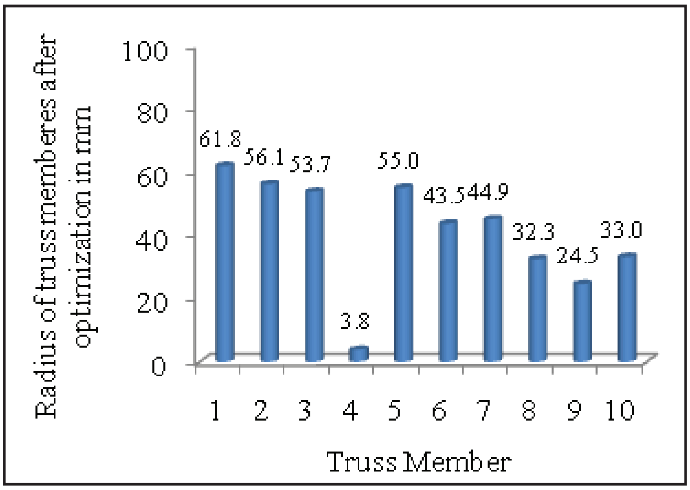

Fig. 13: Radius of the Optimum Members

P. Ratio of minimum weight to maximum stiffness

Weight is reduced by 5.34 times with existing truss, where as maximum stiffness is reduced by 4.32 times only in limits to design constraints 1.76 times margin with maximum stiffness and 1.36 margins with maximum stress.

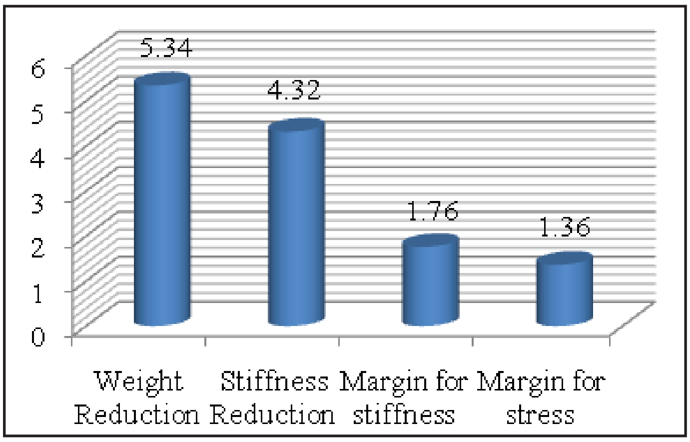

Fig. 15: Margin for Stiffness & Stress

Maximum stiffness to weight ratio is calculated for pre and post optimum ten bar truss. It is identified stiffness to weight ratio is increased by 22% after optimization.

Table 6: Stiffness to Weight Ratio Calculations

Pre-Optimization			Post-Optimization		
Weight (kg)	Stiffness N/m	Stiffness to Weight Ratio	Weight (kg)	Stiffness N/m	Stiffness to Weight Ratio
2563	68.34	2.66%	480	15.6	3.25%

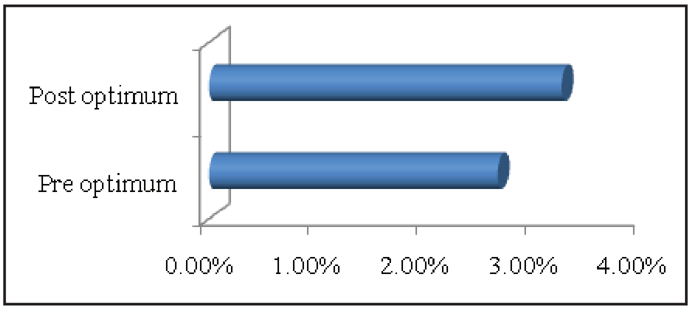

Fig. 16: Margin for Stiffness & Stress

VI. Optimum Truss

Construction of 9-bar truss is recommended for minimum weight and maximum stiffness to weight ratio. It is identified that there is one zero force member in 10 bar truss.

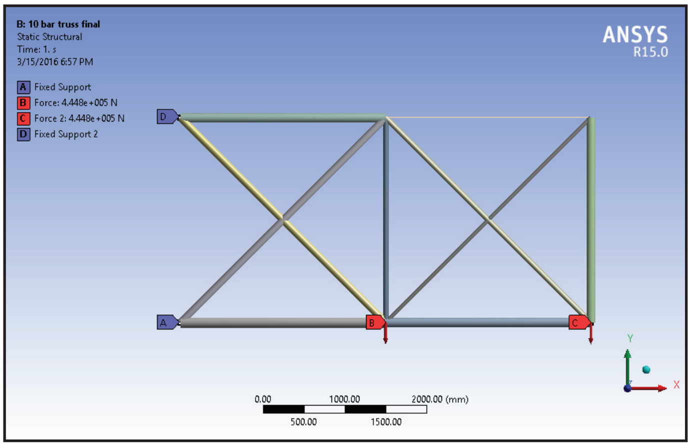

Fig. 17: Optimum Nine Bar Truss for Fabrication

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