Comparison of Measurement Uncertainty Values of Fatigue Tests through Monte Carlo and Kragten Methods

[1]Lisiane Trevisan, [2]Daniel Antonio Kapper Fabricio, [3]Afonso Reguly
[1]IFRS – Campus Farroupilha, Rio Grande do Sul, Brazil
[2]IFSC – Campus Chapecó, Santa Catarina, Brazil
[3]Laboratório de Metalurgia Física – LAMEF /UFRGS, Rio Grande do Sul, Brazil

Abstract
Measurementuncertainty may be determined by various mathematical methods described in literature. The Guide to the Expression of Uncertainty in Measurement (GUM) is the most commonly used method for that. Fatigue testing is important to evaluate material behavior when subjected to cyclic efforts. Thus, this paper aims to compare measurement uncertainty results obtained in fatigue tests from two mathematical methods alternative to GUM: Monte Carlo method (MCM) and Kragten. Measurement uncertainty values were obtained for the parameter 'corrected stress' in fatigue test by both methods. Values obtained in this work from both methods were close to each other, which validates the results and confirms the implementation of alternative methods, such as Kragten and MCM, for fatigue and other mechanical tests.

Keywords
Fatigue Test, Measurement Uncertainty, Monte Carlo, Kragten

I. Introduction

Fatigue is a localized process which causes permanent structural modification in a material, when it is subjected to cyclic stress and deformation. These loading cycles may cause total failure of the component [1]. Most daily components are subjected to fatigue efforts, such as buildings, bridges, car and aircraft parts, as well as machined components [2-3].

Product certification has become an indirect imposition to the world market. The product conformity certification is demonstrated through specific technical standards for each component and its use. Those standards require the documentation of production processes procedures using mainly control of process variables. For each process variable, reliability of the measurements is required, which is performed by trained technicians.

No measurement may be performed without an associated error [4]. Such errors occur due to various problems, such as imperfection of the instrument, reading errors and errors in the procedures used in this measurement. In addition to these problems, the tests carried out in laboratory have controlled environment and conditions, which is not possible in industrial environment, because numerous variables individually or collectively influence the behavior of the material front efforts [5].

Uncertainty is considered a deterministic parameter, that is, a calculated value. If an input variable has its value represented by a probability distribution, their behavior is probabilistic. The Monte Carlo method uses mathematical models involving probabilistic events [6].

The word uncertainty means 'doubt', and it is related to the validity of the result of a measurement. It can be described as a non-negative parameter characterizing the dispersion of the values attributed to a measurand based on the information used, according to the International Vocabulary of Metrology - VIM [7]. Otherwise, the uncertainty of a measurement result reflects the lack of exact knowledge of the value of the measurand [8].

Several factors justify the calculation of measurement uncertainty in the field of testing and calibration. Through the uncertainty value, it is possible to verify the reliability of the obtained results, allowing the approval of these results within the tolerance limit. The fatigue test is a destructive mechanical testing which requires a great time of experimental development, a large number of specimens, and causes a high cost to obtain results with statistical reliability.

It is possible to determine uncertainty using computational methods based on numerical routines. In general, the distribution and propagation of uncertainty start from a mathematical model with data and mathematical constants used in this calculation. This model may seriously affect the confidence of the results and decision-making based on these same results [9]. In this article, numerical routines were developed in Matlab software.

The main objective of this paper is to calculate the measurement uncertainty of material stress through the use of the mathematical methods of Monte Carlo and Kragten for three different levels of roughness for stainless steel AISI 316L. This article will contribute to the knowledge of different mathematical models, and how these values are modified through different roughness levels.

A. Kragten Method

The Kragten method is based on the same methodology of GUM, but it is based on the resolution of partial derivatives numerically resolved [10]. Kragten method has been used in the comparison of measurement uncertainty obtained via Monte Carlo method and for the determination of the largest uncertainty source [11].

It is important to note that this method follows the same recommendations of GUM as is generally used in analytical measurement [12]. Final results are similar as those obtained by GUM if the function is linear and if measurement uncertainty is small when compared to the mean value and, finally, if the measurand is normally distributed [13]. Steps of Kragten method are described in Table 1, where x_1, x_2, x_3 represent uncertainty sources and the standard uncertainty of the source represented by y. The same is applied to the other uncertainty sources.

Table 1: Kragten method steps [12].

$y=f(x_1,x_2,x_3)$	
$y' = f((x_1+u(x_1)),x_2,x_3)$	$\Delta y_1 = \lvert y-y' \rvert$
$y'' = f(x_1,(x_2+u(x_2)),x_3)$	$\Delta y_2 = \lvert y-y'' \rvert$
$y''' = f(x_1,x_2,(x_3+u(x_3)))$	$\Delta y_3 = \lvert y-y''' \rvert$
$u_c = \sqrt{(\Delta y_1)^2+(\Delta y_2)^2+(\Delta y_3)^2}$	

For the implementation of Kragten methodology, the value of standard measurement uncertainty of is approximated by the standard deviation of each source measurement. Note that it is an approximation, and Kragten follows the same method of GUM, which considers standard deviation of measurements, equipment uncertainty, among other sources.

B. Monte Carlo Method

Monte Carlo method (MCM) consists on a technique of artificial sampling which operates numerically complex systems which contain independent input variables. This method can be used in various fields, such as enterprise administration and economics, investment and risk analysis, engineering, and in the determination of measurement uncertainty of tests and calibration [14-15].

The following steps are required to carry out a Monte Carlo simulation: problem formulation, data collect, identification of random variables which will be simulated and their probability distribution and, finally, the execution of the simulation [16]. Measurement uncertainty calculation via MCM is based on the Probability Density Function (PDF) approach.

Fig. 3 illustrates the propagation of probability distributions by MCM. The measurement model Z is a function of three input variables (X_1, X_2, X_3) with the following PDF: p(X_1) (rectangular), p(X_2) (normal) and p(X_3) (triangular). The propagation of these distributions results in a non-symmetric PDF for Z, p(Z). From p(Z), it is possible to obtain the measurement uncertainty associated to the measured value of Z.

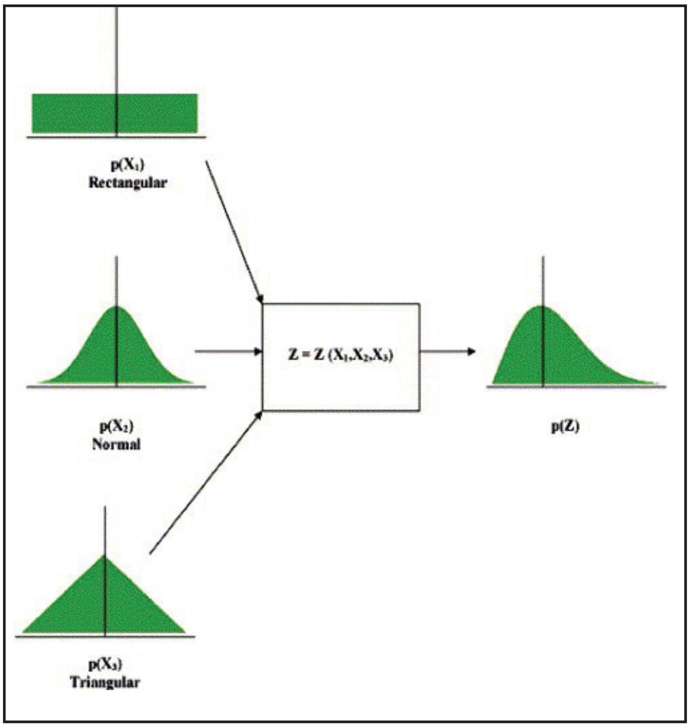

Fig. 1: Propagation of Distribution Through MCM [17].

II. Method used

The metallic material used in this work is a stainless steel SAE 316 L, due to its qualities, such as dimensional stability and excellent resistance to corrosion. The SAE 316 L steel has a low carbon content (maximum of 0.03%) when compared to SAE 316 (maximum of 0.08%) [18]. Dimensions of the specimens are described in Figure 2 [19], with tolerances described by NBR ISO 2768-1 [20].

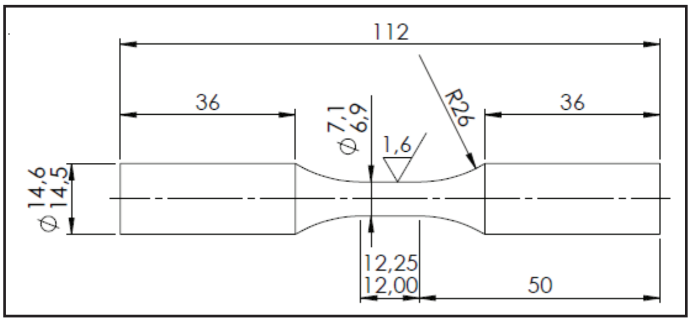

Fig. 2: Dimensions of Test Specimens, in Millimeters.

Experiments were carried out for nine different conditions (stress and roughness levels), according to Fig. 3 with four test specimens for each condition, totalizing 36 test specimens. Then, the mean value and standard deviation of the number of cycles was calculated for each condition.

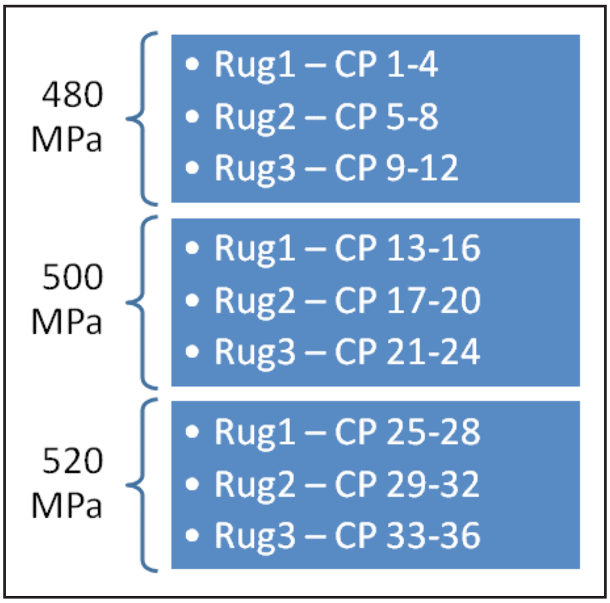

Fig. 3: Experimental Conditions Considered in this Work

Before fatigue tests execution, misalignment verification of test specimens fixture system was carried out. Calibration of static load measurement was also verified for the test machine [17, 22]. Fig. 4 presents the instrumented test specimen fixed in the monitoring system through hydraulic grips.

Fig. 4: Fixing System of the Specimens With Hydraulic Grips

 ISSN : 2249-5762 (Online) | ISSN : 2249-5770 (Print)

Verification was carried out using three combinations of frequency and loading, and results showed the hydraulic grips fixing system has an acceptable error, according to the calibration certificate of testing machine. For the misalignment calculation, ASTM E1012:2012 [23] standard was used. For static calibration, ASTM E467:2008 [24] and BS 7935-1:2004 [25] standards were followed.
Roughness measurements were carried out with a profilometer in the region of interest. Five measurements for each test specimen were taken with 0.25 mm spacing between them. In order to minimize error in the measurement, a fixing system was used. The three roughness levels were classified as described in Table 2.

Table 2: Test Specimens Classification According to Roughness Level

Roughness	Surface preparation
$Ru1_{\#800p}$	Sanding with sandpaper #800 and polishing with diamond paste 6 microns
$Ru2_{\#500}$	Sanding with sandpaper #500, free polishing
$Ru3_{\#320}$	Sanding with sandpaper #320, free polishing

Brinell hardness testing allows minimizing measurement error, since it considers the penetrator diameter and surface imprint in its calculation. A semiautomatic durometer with a 2.5 mm diameter sphere and 187.5 kgf loading was used. Calibration was carried out with a 216 HB standard, considering an expanded measurement uncertainty of 10.3 HB at a confidence level of 95%. Five hardness measures were carried out for each test specimen.
Fatigue tests were carried out on a servo-hydraulic universal machine with loading capacity of 100 kN. Test parameters were as follows: frequency of 20 Hz, fatigue ratio of 0.1, runout of 1 million cycles and as failure criterion the total rupture of test specimen.
In this work, procedures for measurement uncertainty calculation for Monte Carlo and Kragten method were developed in Matlab® software. Uncertainty calculation was carried out by both method for the parameter 'corrected stress', according to Equation (1).

$$\sigma_{corrigida} = C_{\sigma,R} . \sigma_{ensaio} \quad (1)$$

Corrected stress is described by FKM-Guideline [26]. This parameter represents the testing stress considering the influence of surface roughness during the test, adjusted by a correction factor ($C_{\sigma,R}$), described by Equation (2).

$$C_{\sigma,R} = 1 - a_r \log(R_z).\log(\frac{2S_{t,u}}{S_{t,u,\min}}) \quad (2)$$

The values of the correction fator are widely described in FKM-Guideline [26].

III. Results and conclusions

For the measurement uncertainty through MCM, the values of lower limits (L_i) and upper limits (LS) are required for each variable, according to the mean value (μ) and standard deviation (σ) [27], according to Equations (3) and (4).

$$-1.96 = \frac{L_i - \mu}{\sigma} \quad (3)$$

$$1.96 = \frac{L_S - \mu}{\sigma} \quad (4)$$

Thus, measurement uncertainty for a confidence level of 95% and K = 2 may be described by Equation (5) [27].

$$U(p = 95\%) = (L_S - L_i)/2 \quad (5)$$

For the analysis through Kragten method, four simulations for each test condition were carried out, as described in Tables 3, 4 and 5.
Simulation 'A' presents the value of the corrected stress ($\sigma_{corrected}$) obtained from the mean values of each uncertainty source, named 'Referential'. Simulation 'B' was carried out by varying the roughness level (as described in Equation 4), and its value is represented by the sum of the mean value and its respective standard deviation.

$$x_1 = \bar{x} + \sigma \quad (4)$$

This same methodology was applied for simulations 'C' and 'D', where hardness and diameter, respectively, were varied. Simulations 'B', 'C' and 'D' are compared to simulation 'A', which is represented by the values of $\Delta\sigma_{corrected}$. Thus, it is possible to evaluate which uncertainty source represents the largest influence in the corrected stress ($\sigma_{corriected}$) value.
According to Table 3, roughness is the largest uncertainty source for testing stress of 480 MPa, which is verified by the values for simulation 'B'.

Table 3: Simulation for Test Stress of 480 MPa.

Condition	Simulation	$\sigma_{corrected}$	$\Delta\sigma_{corrected}$
$Ru1_{\#800p}$	A	489.58	Referential
	B	487.64	1.94
	C	489.83	0.25
	D	489.44	0.14
$Ru2_{\#500}$	A	481.22	Referential
	B	480.28	0.94
	C	481.23	0.01
	D	481.07	0.15
$Ru3_{\#320}$	A	472.49	Referential
	B	471.28	1.21
	C	471.99	0.50
	D	472.23	0.27

Table 3 shows the roughness is the most significant uncertainty source for testing stress of 480 MPa. Furthermore, values found by Kragten method are close to those found by MCM for combined measurement uncertainty. It is important to note for roughness Ru2#500 the influence of uncertainty contribution related to the hardness is small (0.01) when compared to other values, such as 0.25 and 0,50.For roughness conditions Ru1#800p and Ru3#320, uncertainty sources with larger influence in the expanded uncertainty of corrected stress are: diameter, hardness and roughness, respectively.

As for the test stress of 500 MPa, the values obtained are shown in Table 4.

Table 4: Simulation for Test Stress of 500 MPa.

Condition	Simulation	$\sigma_{corrected}$	$\Delta\sigma_{corrected}$
$Ru1_{\#800p}$	A	502.07	Referential
	B	499.26	2.81
	C	502.12	0.05
	D	501.93	0.14
$Ru2_{\#500}$	A	493.65	Referential
	B	492.10	1.55
	C	493.53	0.12
	D	493.37	0.28
$Ru3_{\#320}$	A	489.19	Referential
	B	486.90	2.29
	C	488.61	0.57
	D	488.91	0.28

According to Table 4, roughness is the largest uncertainty source for testing stress of 500 MPa. It is important to note this influence is more significant for condition Ru1#800p and Ru3#320, thus, this difference is shown between values obtained in Table 3 and 4. For the condition Ru3#320, hardness presents a large contribution for measurement uncertainty when compared to other conditions.
Finally, Table 5 presents results obtained by Kragten method for testing stress of 520 MPa.

Table 5: Simulation for Test Stress of 520 MPa.

Condition	Simulation	$\sigma_{corrected}$	$\Delta\sigma_{corrected}$
$Ru1_{\#800p}$	A	525.61	Referential
	B	524.71	0.90
	C	525.64	0.03
	D	525.31	0.30
$Ru2_{\#500}$	A	511.12	Referential
	B	508.38	2.74
	C	511.11	0.01
	D	509.13	1.99
$Ru3_{\#320}$	A	505.47	Referential
	B	503.92	1.55
	C	505.22	0.26
	D	504.18	1.29

As verified in Table 5, roughness is also the most significant uncertainty source for testing stress of 520 MPa. Slight differences were found between values calculated by MCM and Kragten, and the largest difference was represented by condition Ru2#500.
For standard measurement uncertainty values calculation via Monte Carlo method, correction factor ($C_{\sigma,R}$) was calculated from roughness and hardness values obtained experimentally for five measurements in each test specimen. A normal (Gaussian) probability distribution was considered for each uncertainty source. Thus, starting from the mean value and standard deviation of correction factor, this value is multiplied by the stress value used in fatigue test (480 MPa, 500 MPa and 520 MPa), obtaining the value of the corrected stress.
Uncertainty values obtained by Kragten and MCM are presented in Table 6, compared for each value of stress and roughness level.

Table 6: Measurement Uncertainty Values Obtained by Kragten and MCM

Stress	Roughness	Kragten	MCM
480 MPa	$Ru1_{\#800p}$	1.96	2.20
	$Ru2_{\#500}$	0.95	0.99
	$Ru3_{\#320}$	1.34	1.31
500 MPa	$Ru1_{\#800p}$	2.81	3.24
	$Ru2_{\#500}$	1.58	1.67
	$Ru3_{\#320}$	2.38	2.62
520 MPa	$Ru1_{\#800p}$	0.95	0.98
	$Ru2_{\#500}$	3.38	3.73
	$Ru3_{\#320}$	2.03	2.11

According to Table 6, expanded measurement uncertainties for both methods are similar to each other, varying between -2,2% and 15,3%. However, depending on the project exigence, this difference may be significant. The choice of the best method to be used depends on the analyst. As MCM presented larger values for uncertainty, it seems to be a more conservative method.
Finally, a statistical analysis of uncertainty results was carried out via Analysis of Variance (ANOVA), at a confidence value of 95%. Table 7 presents ANOVA results, were DF represents the degrees of freedom, SS the sum of squares, MS the mean squares and p-value the significance value of the factor.

Table 7: Analysis of variance for results.

Source	DF	SS	MS	F	p-value
Stress	2	2.87	1.44	89.35	0.00
Roughness	2	0.02	0.01	0.71	0.55
Method	1	0.12	0.12	7.47	0.05
Stress*Roughness	4	10.02	2.51	155.91	0.00
Stress*Method	2	0.02	0.01	0.68	0.56
Roughness*Method	2	0.01	0.01	0.44	0.67
Error	4	0.06	0.02		
Total	17	13.13			

ANOVA was carried out considering main factors: Stress, Roughness level and Method (for the calculation of measurement uncertainty). The interactions Stress X Roughness, Stress X Method and Roughness X Method were also considered. The response variable was the uncertainty value, according to Table 6.
Fig. 5 presents main effects plot for the ANOVA, and Fig. 6 shows interaction plot for uncertainty.

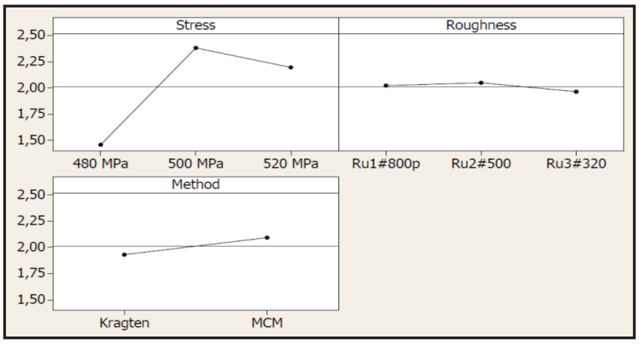

Fig. 5: Main Effects Plot for Uncertainty.

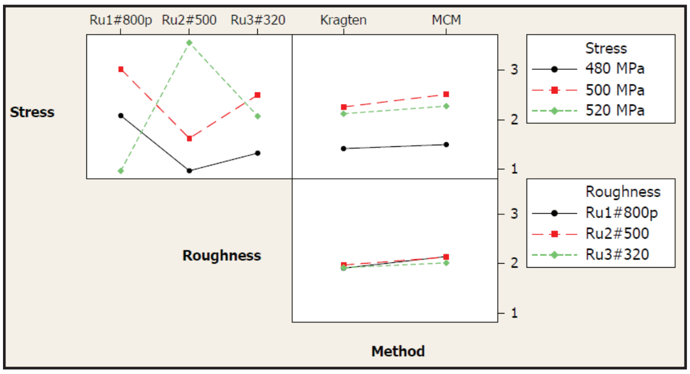

Fig. 6: Interaction Plot for Uncertainty

According to ANOVA results, the method of calculation of measurement uncertainty was not significant and neither were the interactions Stress X Method and Roughness X Method.
This implies the method of calculation for measurement uncertainty (Kragten or Monte Carlo) does not affect significantly uncertainty results. This validates the results and confirms the implementation of alternative methods, such as Kragten and MCM, for fatigue and other mechanical tests.

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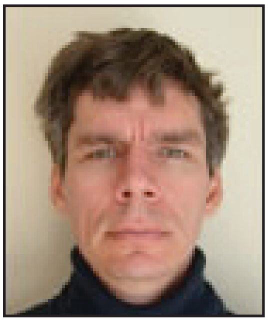
Lisiane Trevisan received degree in Metallurgical Engineering from the Federal University of Rio Grande do Sul - UFRGS (2005), Master in science in PPGE3M/UFRGS (2008). PhD in Mining, Metallurgical and Materials -PPGE3M/UFRGS. She is currently Professor of Basic, Technical and Technological Education at the Federal Institute of Education, Science and Technology of Rio Grande do Sul - IFRS. She has experience in Materials and Metallurgical Engineering area with emphasis on casting, mechanical testing and metrology.

Daniel Antonio Kapper Fabricio is Industrial Engineer by Federal University of Rio Grande do Sul (2013) and Master in Science and Technology of Materials in PPGE3M/UFRGS (2015), working as Professor at Federal Institute of Santa Catarina. Currently pursuing PhD in Science and Technology of Materials by PPGE3M/UFRGS.



AfonsoReguly holds a degree in Metallurgical Engineering from the UFRGS (1992), master's degreein Mining, Metallurgical and Materials Engineering at PPGE3M/UFRGS (1994), PhD in Mining, Metallurgical and Materials Engineering by PPGE3M/UFRGS (1999) and post-doctorate at Colorado School of Mines (2012). He is currently Associate Professor at the Federal University of Rio Grande do Sul - UFRGS. He has experiencein the area of Materials and Metallurgical Engineering, withemphasis on Transformation Metallurgy.