Flexural Behavior of Functionally Graded Sandwich Plates

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Abstract

In the present article, the flexural responses of the functionally graded (FG) sandwich plates under different support conditions and subjected to sinusoidal load are investigated. The FG sandwich plates are assumed to be made in such a way that the top and bottom layer are metal rich and the core layer is made up of homogenous and isotropic material which is generally ceramic. The desired responses have been computed using the commercial finite element package (ANSYS). The FG plate model has been developed with the help of ANSYS parametric design language (APDL) code based on the first order shear deformation theory (FSDT). For discretization purpose, SHELL281 type element with eight node and having six degrees of freedom at each node has been employed from the ANSYS elemental library. The convergence behaviour of the present simulation results has been checked by solving different numerical examples. Then, the flexural responses are computed using the present simulation model and compared with earlier published results available in open literature to show its validity. Finally, the effect of power law indices, different FGM properties (aluminum/alumina and aluminum/zirconia), aspect ratios and type of sandwich symmetry on the bending responses of FG sandwich plates have been investigated through various numerical illustrations and discussed in details.

Keywords

FG Sandwich Plate, Flexural Response, FSDT, ANSYS.

I. Introduction

In recent years, laminated composites are mostly used in various engineering structures, aerospace and automotive industries as they are light in weight and at the same time possess higher strength/stiffness to weight ratio. Usually, the conventional laminated composite structures are made up of different layers of homogenous lamina which are bonded together so as to gain increased mechanical properties. Since there is abrupt change in material properties between the interfaces of different materials high inter-laminar stresses are caused and results in the failure of the structure. This phenomenon is regarded as delamination of the laminated structure. In order to overcome the problem of delamination the Functionally Graded Materials (FGMs) are used in which the properties vary smoothly and continuously along the thickness direction. In general, the FGMs are constructed by ceramic and metal, in such a way that properties are varied by changing volume fraction of constituent materials along thickness of plate.

II. Literature Review

Various studies have been performed to investigate the behavior of functionally graded materials. Nguyen et al. [1] (2008) studied the bending stresses in clamped FG plates and cylindrical shells using a model based on the FSDT in association with energy equivalence method. Bayat et al. [2] (2009) presented the semi analytical solutions for the bending response of simply supported FG rotating disks based on the FSDT. Alieldin et al. (2011) [3] proposed a transformation approach based on the FSDT to investigate the mechanical behavior of FG plates. Thai and Choi (2013) [4] presented a simple FSDT with four unknowns for the bending and free vibration analysis of FG plates. Castellazzi et al. (2013) [5] presented the nodal integrated finite element method (FEM) to study the static behavior of FG plates based on the FSDT. Ardestani, et al. (2014) [6] analyzed simply supported and clamped FG stiffened plates based on the FSDT Using reproducing kernel particle method (RKPM). Yu et al. (2015) [7] presented a novel simple FSDT-based isogeometric analysis (IGA) for geometrically nonlinear analysis of FGM plates. Zenkour (2005) presented the bending analysis of simply supported FG sandwich plates using sinusoidal, third-order, first-order and classical shear deformation theories. Recently, Mantari and Granados (2015) [9] presented a static analysis of FG sandwich plates by using a new FSDT.

It is evident from the brief review that, many efforts have already been made in past for the theoretical developments for the numerical or analytical solutions of linear/nonlinear flexural behaviour of FGM sandwich plates. The aim of the present investigation is to study the bending behavior of FG sandwich flat plates under the sinusoidal loads. In order to do so, a simulation model for the FG sandwich plate has been developed using APDL code and discretized using Shell 281 element from ANSYS elemental library. Numerical illustrations are presented to show the effect of various parameters on the flexural response of simply supported and clamped FG sandwich plates and discussed in detail.

III. Methodology

The geometry and coordinate of a typical FG sandwich panel of length "a", width "b" and thickness "h" considered for the present analysis are shown in fig. 1(a) and (b), respectively. The panel is modeled in ANSYS 15.0 APDL environment that assumes a displacement field in the framework of the FSDT. Thus, the displacements at any point in the panel u, v and w along x, y and z directions, respectively can be represented as:

$$\begin{aligned} u(x,y,z) &= u^0(x,y) + z\theta_X(x,y) \\ v(x,y,z) &= v^0(x,y) + z\theta_y(x,y) \\ w(x,y,z) &= w^0(x,y) + z\theta_z(x,y) \end{aligned} \qquad (1)$$

where, u^0, v^0 and w^0 are the displacements of any point in the mid-plane along x, y and z directions, respectively and θ_x, θ_y and θ_z are the shear rotations.

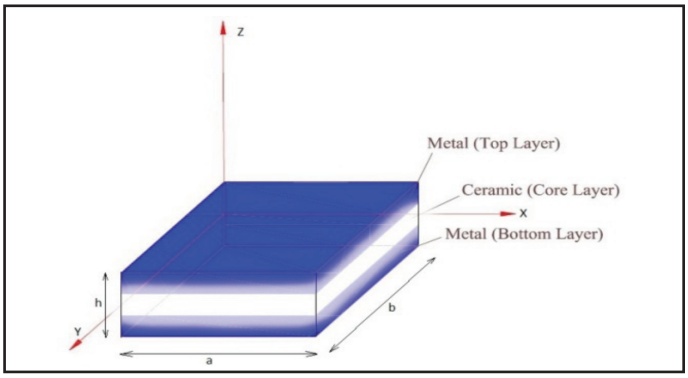


Fig. 1(a): Geometry and Dimensions of FG Sandwich Plate

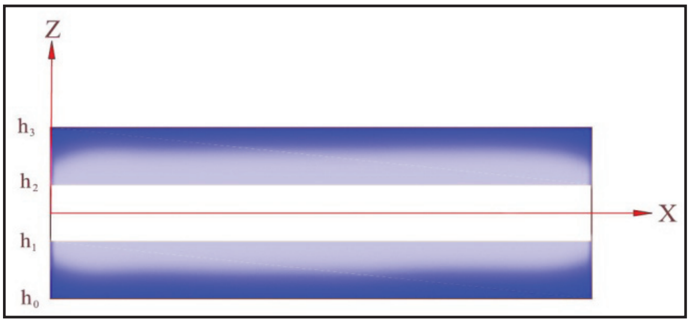


Fig. 1(b): Coordinates of FG Sandwich Plate

In ANSYS mechanical APDL, various element types are available for modeling of layered structures. For the present analysis, Shell 281 element has been taken. Shell 281 is known for its robustness and suitability for the analysis of thin to slightly thick shell structures. The considered element has total eight nodes having six degrees of freedom per each node say, translation along x, y, and z axes, and rotation about x, y, and z axes. This element gives satisfactory results for linear, large rotation, and/or large strain nonlinear solutions. This element accounts for load stiffness effects of distributed pressure. For the present modeling purpose, formulation is based on the FSDT and logarithmic strain and true stress measures. Figure 2 represents the geometrical parameters such as node location and element coordinate system for this element. The element is defined by shell section information and by eight nodes (I, J, K, L, M, N, O and P).

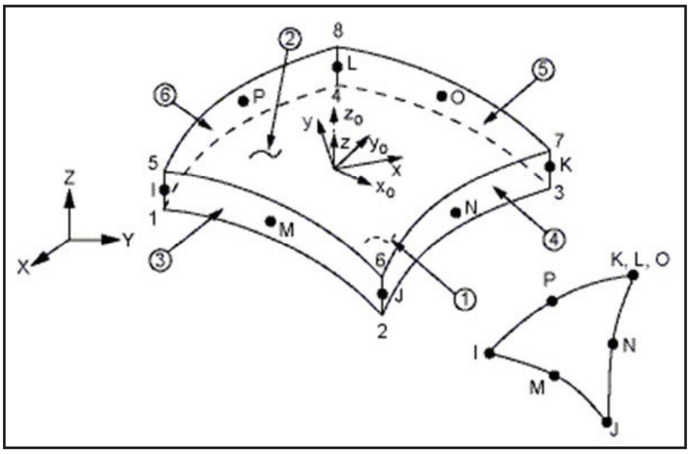


Fig. 2: Shell 281 Geometry (ANSYS 15.0)

In the FG sandwich plate, the material properties vary in the thickness direction. The coordinates of different layer are denoted as:

For, $Z = h_0$, Metal rich
$Z \in (h_1, h_2)$, completely ceramic
$Z = h_3$, Metal rich

In this sandwich plate the face layers are made up of isotropic material and their properties vary in the direction of thickness (z-direction) and the core layer is completely homogenous and isotropic (ceramic). It is assumed that the Poisson's ratio remains constant and the Young's modulus vary as,

$$E(z) = E_m + (E_c - E_m)V^{(n)} \quad (2)$$

where, E_m and E_c are the Young's Modulus of elasticity of the metal and the ceramic, respectively, $V^{(n)}$ is the volume fraction, n=1, 2, 3. The volume fraction varies through the thickness as per the following power law:

$$V^{(1)} = \left(\frac{x_3 - h_0}{h_1 - h_0}\right)^k, z \in [h_0, h_1]$$
$$V^{(2)} = 1, z \in [h_1, h_2] \quad (3)$$
$$V^{(3)} = \left(\frac{x_3 - h_3}{h_2 - h_3}\right)^k, z \in [h_2, h_3]$$

where, k = power law index

In order to obtain the flexural responses, a sinusoidal load is applied on the surface of the FGM sandwich plate represented as follows [8]:

$$q(x, y) = q_0 \sin(\lambda x)\sin(\mu y) \quad (4)$$

where, q_0 = intensity of load at the center of panel, $\lambda = \pi/a$ and $\mu = \pi/b$.

In the present study, various kinds of symmetry of the FGM sandwich plate are used and denoted as: 1-1-1, 1-2-1, 2-1-2 and 1-0-1. Considering the thickness of plate as "h" and the plane of symmetry to be at the mid-surface of plate, the arrangements of the core and the face sheets for each of the aforementioned symmetries are as follows [8]:

1-1-1: The thicknesses of all the layers (top, core and bottom) are equal. The coordinates of layer are h_0 =-h/2, h_1=-h/6, h_2=h/6, and h_3=h/2.

1-2-1: The sum of face thickness is equal to the core layer thickness of the FGM sandwich. The coordinates of the layers are h_0 =-h/2, h_1=-h/4, h_2=h/4, and h_3=h/2.

2-1-2: The thickness of the core layer is equal to half the thickness of face. The coordinates of layer are h_0 =-h/2, h_1=-h/10, h_2=h/10, and h_3=h/2.

1-0-1: The core layer is absent and the sandwich is made of only two layer. The coordinates of layers are h_0 =-h/2, h_1=0, h_2=0, and h_3=h/2.

IV. Results and Discussions

The bending responses of FG sandwich plate have been computed using the present simulation model. The properties of different materials considered for the computation purpose are provided in Table 1. Sinusoidal distributed load acting on the surface of the panel has been considered with intensity of the load at the plate center as "q_0 (MPa)". In the present analysis, two different boundary conditions have been employed to restrict the rigid body motion and to reduce the number of unknowns for finding the solution. The boundary conditions are all sides simply supported (SSSS) and all sides clamped (CCCC) and given as:

A. All Sides Simply Supported (SSSS)

$v^0 = w^0 = \theta_y = \theta_z = 0$ at x = 0 and a;
$u^0 = w^0 = \theta_x = \theta_z = 0$ at y = 0 and b.

All sides clamped (CCCC)

$u^0 = v^0 = w^0 = \theta_x = \theta_y = \theta_z = 0$ at both x = 0 and a; y = 0 and b.

Table 1: Properties of Metal and Different Ceramic Materials Used for Computation Purpose.

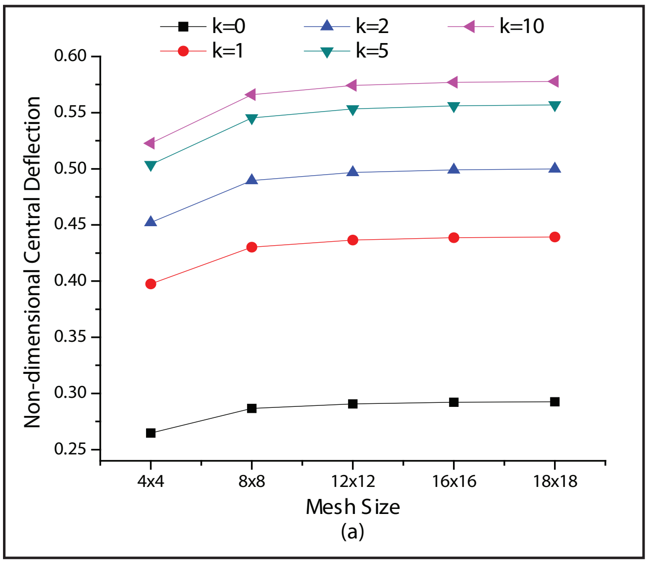
S. No.	Material	Young's Modulus	Poisson's Ratio
1	Aluminium	70GPa	0.3
2	Zirconia	151GPa	0.3
3	Alumina	380GPa	0.3

A. Convergence Study

The bending responses for FGM sandwich plates under sinusoidal distributed loads are computed over various mesh refinement under SSSS and CCCC support conditions. For the computation purpose, two combinations of materials as metal and ceramic have been used namely, Aluminium-Zirconia and Aluminium-Alumina, and plotted in fig. 3 (a) and (b), respectively. The properties of the materials are as shown in Table 1. The results in fig. 3 clearly depict that the responses computed using the present simulation model are converging well with mesh refinement. Based on the convergence study a 16×16 mesh has been chosen for determining the responses further throughout the present study.

B. Comparison Study

In order to validate the present simulation model, an Aluminium-Zirconia sandwich plate as in Zenkour (2005) [8] has been considered. The bending responses are computed for three different value of power law index (k= 0, 1, and 5) with three different symmetries (1-0-1, 2-1-2, and 1-2-1). For the computation purpose, the geometry, material properties, support conditions and non-dimensional formula for the central deflection used are similar to the reference [8]. The current results and the reference values are shown in Table 2, which show that the present results are as good as the reference values. The slight difference between the results is due to the use of different theories for the modeling purpose.



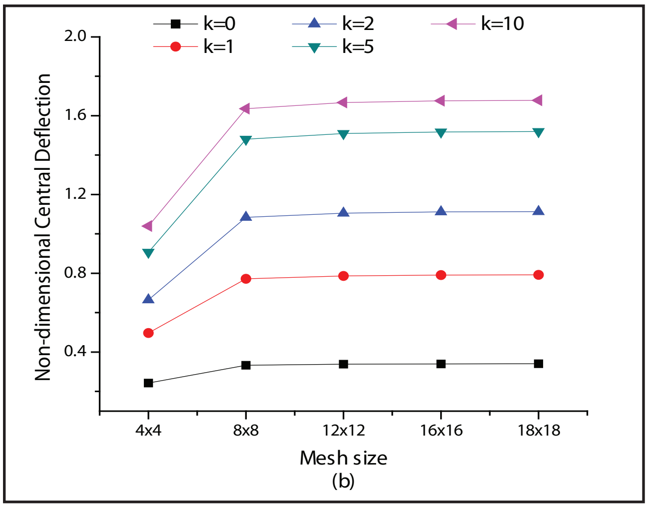


Fig. 3: Convergence Behaviour of Non-Dimensional Central Deflection of FG Sandwich Plate (a) Aluminium-Zirconia (for q_0 =150MPa) (b) Aluminium-Alumina (for q_0 =1MPa).

Table 2: Comparison of Non-Dimensional Bending Responses of FGM Sandwich Plate Under Sinusoidal Distributed Load

Power Index k	Theory	Non-Dimensional Central Deflection		
		Type of Symmetry		
		1-0-1	2-1-2	1-2-1
0	TSDT[8]	0.19606	0.19606	0.19606
	Present	0.19270	0.19270	0.19270
	% Difference	-1.74364	-1.74364	-1.74364
1	TSDT[8]	0.32360	0.30630	0.27090
	Present	0.32120	0.30450	0.26990
	% Difference	-0.7472	-0.59113	-0.37051
5	TSDT[8]	0.41120	0.39418	0.35123
	Present	0.40420	0.38790	0.33250
	% Difference	-1.73182	-1.61897	-5.63308

C. Numerical Illustrations

The ability and accuracy of the present simulation model have been demonstrated in the convergence and comparison study. Now, in this section some new examples are solved using the present model to bring out the quantitative understanding of various parameters on the flexural behavior of FG sandwich plates. The effects of power law index, type of symmetry and aspect ratio on the central deflection of the FGM sandwich plate have been investigated. For the analysis purpose, the material properties used are as in Table 1 and the central deflection values are expressed in non-dimensional form as $W_{non\text{-}dimensional} = W/h$.

1. Effect of Power Index

The material properties of the FG sandwich plate vary continuously through the thickness and the variation pattern is governed by the power law index. To study the effect of power index on the flexural responses first, a simply supported (SSSS) Aluminium-Alumina FG sandwich plate (h=0.01m, a/h=10, a/b=1 and symmetry type 1-2-1) with six different values of power index (k= 0, 0.5, 1, 2, 5 and 10) and subjected to sinusoidal distributed load of various peak value (q_0 = 0, 4, 8, 12, 16, 20MPa) has been considered. The results are plotted in fig. 4 (a). Then, all sides clamped (CCCC) Aluminium-Alumina and Aluminium-Zirconia FG sandwich plates for both symmetric (1-1-1) and anti-symmetric (2-1-1) geometry subjected to constant sinusoidal distributed load (q_0 =100MPa) has been considered. Deflection responses are computed for different values of power index (k= 0, 0.5, 1, 2, 4, 5, 8, 10) and shown in fig. 4 (b). It is observed that as the value of power index increases, the central deflection also increases. It is also seen that under a given load, Aluminium-Zirconia sandwich panel has greater central deflection values than the Aluminium-Alumina sandwich. The central deflection of anti-symmetric sandwich panel is more than the symmetric sandwich panel.

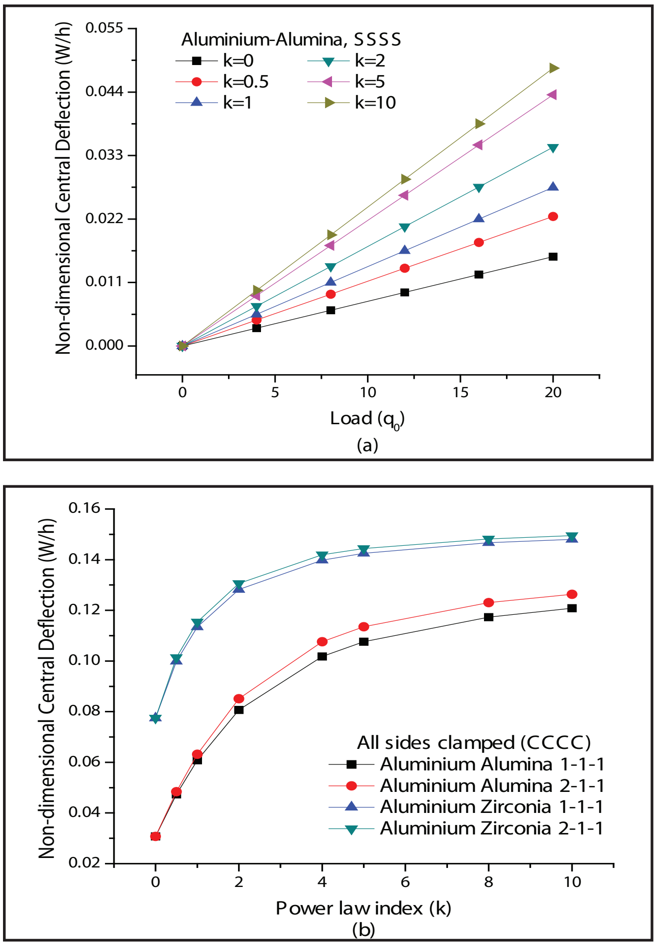


Fig. 4: Effect of power index (k) on the non-dimensional central deflection of FG sandwich plate.

2. Effect of Type of Symmetry

In order to study the effect of type of symmetry on the bending behavior of FG sandwich plate, two combinations of materials, Aluminium-Alumina and Aluminium-Zirconia, has been considered. The FG sandwich panels (h=0.01m, a/h=10, a/b=1, k=2 and SSSS) of four types of symmetries (1-0-1, 1-1-1, 1-2-1 and 2-1-2) are subjected to sinusoidal distributed load (q_0 = 10, 20, 30, 40 and 50 MPa). The results presented in fig. 5 depict that the symmetric sandwich panel with 1-0-1 and 1-2-1 pattern exhibit highest and least central deflection irrespective of materials used.

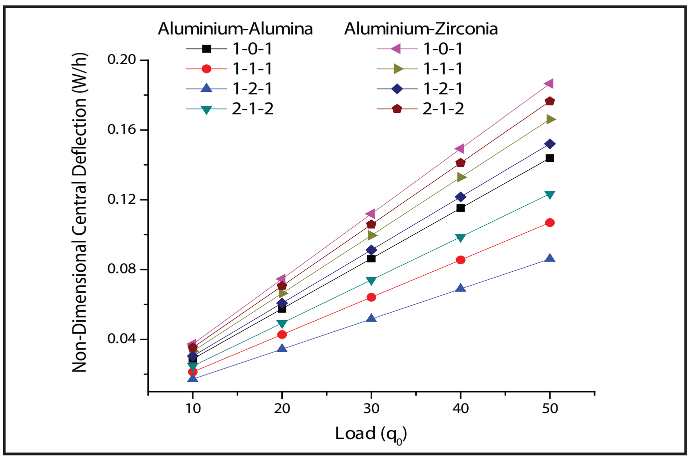


Fig. 5: Effect of Type of Symmetry on the Non-Dimensional Central Deflection of Aluminium-Alumina and Aluminium-Zirconia FG Sandwich Plate.

3. Effect of Aspect Ratio

The effect of aspect ratio on the non-dimensional central deflection has been investigated by considering all sides clamped Aluminium-Alumina FG sandwich plate (h=0.01m, a/h=20, k=5, 1-0-1) subjected to sinusoidal distributed load (q_0 = 4, 8, 12, 16 and 20MPa). The responses are computed using five different aspect ratios (a/b = 0.5, 1, 1.5, 2 and 2.5) and shown in fig. 6. It can be clearly seen from the results that the non-dimensional central deflection decreases with increasing aspect ratio.

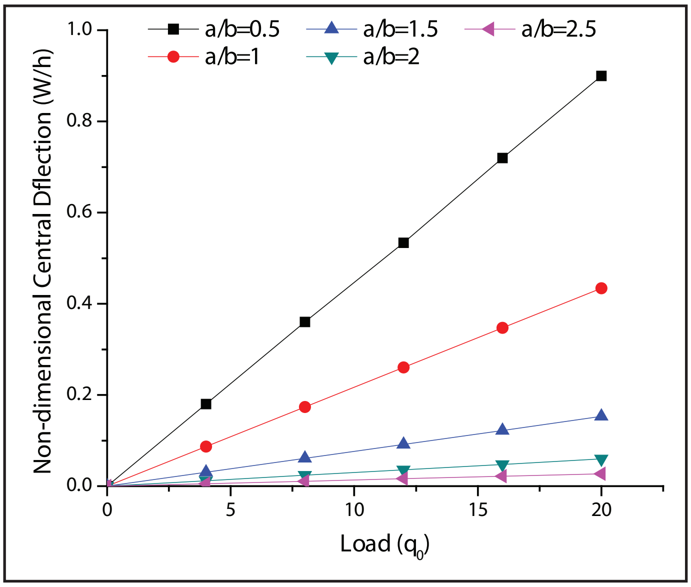


Fig. 6: Effect of Aspect Ratio on the Non-Dimensional Central Deflection in FG Sandwich Plate

V. Conclusions

The flexural behavior of functionally graded sandwich plates under uniformly distributed sinusoidal load has been investigated in this article. Each face of the sandwich panel is chosen as a functionally graded material containing two material phases, metal and ceramic. The variation of properties with respect to the thickness is governed by power law, such that the core is homogenous isotropic ceramic material and both top and bottom faces are metal rich. In order to compute the desired responses a simulation model for the FG sandwich plate has been developed in ANSYS 15.0 using APDL code. The convergence and validation of the present simulation model has been established and few new examples are solved to investigate the effect of various parameters on the deflection behavior of the FG sandwich plates. Based on the parametric studies, it is observed that, the central deflection values increase with increasing power index value and decrease with aspect ratios. It is worthy to note that central deflection of anti-symmetric (2-1-1) sandwich panel is more than symmetric (1-1-1) sandwich panel. However, sandwich panels with higher central thickness of ceramic material depict greater stiffness which is expected.

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