

# Static Analysis and Modal Analysis of Composite Mono Leaf Spring By Using ANSYS

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## Abstract

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. Though there are many types of springs, yet the following, according to their shape, are important is given by "Helical springs, Conical and Volute springs, Torsional springs, Laminated (or) Leaf springs, Disc(or) Belleville springs. Leaf springs (also known as flat springs) are made out of flat plates. The advantage of leaf spring over helical spring is that the ends of the spring may be guided along a definite path as it deflects to act as a structural in addition to energy absorbing device. Generally leaf springs are widely used in automobiles. Mainly two types of leaf springs are present, they are Mono leaf springs and Multi leaf springs. A composite is usually made up of at least two materials out of which one is the binding material, also called matrix and the other is the reinforcement material. The advantage of composite materials over conventional materials stem largely from their higher specific strength, stiffness and fatigue characteristics, which enables structural design to be more versatile. ANSYS is general-purpose finite element analysis (FEA) software package. Finite Element Analysis is a numerical method of deconstructing a complex system into very small pieces (of user-designated size) called elements. The aim of our project is to analyze the mono leaf spring for various loads made up of different cross-sections with three different materials i.e., steel, E-glass fiber and carbon fiber. Comparison is done for three materials of different cross-sections of mono leaf springs.

## Keywords

Springs, Leaf Spring, E-Glass, Carbon, Steel

## I. Introduction

A spring function is to distort when loaded and to recover its original shape when the load is removed. Mechanical springs are used in machines and other applications mainly to exert force, to provide flexibility, to store or absorb energy.

### A. Types of Springs

#### 1. Helical Springs

Helical springs are made of wire coiled in the form of helix and are primarily intended for compressive or tensile loads. The cross-section of wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are compression helical spring and tension helical spring. Helical springs are said to be closely coiled, when the helix angle is very small ( $<10^\circ$ ), where as in open coil helical spring the helix angle is large.

**Advantage:** These springs are easy to manufacture. They are available in wide range. They are highly reliable. They have constant spring rates. Their performance can be predicted more accurately. Here characteristics can be varied by changing dimensions.

#### 2. Conical and Volute Springs

The conical and volute spring are special applications where the spring rate increases in increasing load. Another feature of these types of springs is the decreasing number of coils results in an increasing spring rate. This characteristic is sometimes utilized in vibration problems where springs are used to support a body that has varying mass.

#### 3. Torsion Springs

These springs may be of helical or spiral type. Helical types of springs are used where the load tends to wind up the springs and are used in electrical mechanisms. Spiral type is used where the load tends to increase the number of coils and are used in watches and clocks.

#### 4. Laminated or Leaf Springs

The laminated or leaf spring (also known as flat spring) consists of a number of plates (known as leaves) of varying length held together by means of clamps and bolts. These types of springs are most used in automobiles.

#### 5. Disc Springs

These springs consist of a number of conical discs held together by a central bolt or tube. These springs are used in applications where high spring rates and compact spring units are required.

#### (i). Leaf Springs

Leaf springs also known as flat spring or cart spring, made up of flat plates. Today leaf springs are still used in commercial vehicles such as cars, vans and trucks, and railway carriages. For heavy vehicles, they have the advantage of spreading the load more widely over the vehicle's chassis. The importance of leaf spring is to carry bump loads (i.e. due to road irregularities), support the chassis weight, control axle damping, control braking forces, and to provide better suspension. Leaf springs are designed in two ways:

- Multi leaf springs
- Mono leaf springs.

### II. Construction of Leaf Spring

Semi elliptical leaf springs are almost universally used for suspension in light and heavy commercial vehicle. For cars also, these are widely used for rear suspension.

Semi elliptical leaf spring consists of a number of leaves called blades. These blades vary in length. The composite spring is based on the theory of a beam of uniform strength.

#### A. Analysis of Leaf Spring Characteristics

The function of a leaf spring may be analyzed by means of some simple types of beams and their characteristics. Consider a cantilever beam of rectangular cross section, whose width is 'b' thickness 'h' and length 'L', is subjected to a load 'F' at its free end. Due to this load the beam tries to bend and the maximum bending moment at the fixed end is

$$\sigma_{\max} = S_b \frac{M}{Z} = \frac{MY}{I} = \frac{F \cdot l \cdot h/2}{bh^3/12} = \frac{6Fl}{bh^2}$$

$$\sigma_{\max} = \frac{6Fl}{bh^2}$$

Maximum deflection, obtained at the free end is given by,

$$\delta_{\max} = \frac{Fl^3}{3EI} = \frac{F \cdot l^3}{3E \frac{bh^3}{12}} = \frac{4Fl^3}{Ebh^3} = \frac{2}{3} S_b \frac{l^2}{Eh}$$

$$\delta_{\max} = \frac{2}{3} S_b \frac{l^2}{Eh}$$

Where, E Is the Elastic modulus of the spring material. Since the leaf spring is similar to simply supported beam. let us consider a simply supported beam of length of length L(2l) and a central load of W(2F). The width and thickness of plates (or beam) may be 'b' and 'h' respectively. For this case, maximum bending stress induced at the center is

$$\sigma_{\max} = \frac{MY}{I} = \frac{WL \cdot h}{\frac{4}{12} \cdot \frac{b^3}{12}} = \frac{2F \cdot 2l \cdot h}{\frac{4}{12} \cdot \frac{b^3}{12}} = \frac{6Fl}{bh^2}$$

Similarly, maximum deflection at the center is

$$\delta_{\max} = \frac{WL^3}{48EI} = \frac{2F \cdot (2l)^3}{48 \cdot E \cdot \frac{b^3}{12}} = \frac{4Fl^3}{Ebh^3}$$

From the above two analysis we can come to conclusion that the simply supported beam of length L(2l) subjected to a load W(2F) may be treated as Double cantilever beam fitted side by side and loaded at its ends.

**B. Design of Leaf Spring:**

Since the leaf spring is constructed by certain number of full length Leaves and other by graduated length leaves, it is design on the basis of combined strength and deflection characteristics of both full length and graduated leaves. F = Load applied at the end of the spring. F<sub>f</sub> = load shared by full length leaves. F<sub>g</sub> = load shared by the graduated leaves along with master leaf N<sub>g</sub> = total number of leaves. N<sub>f</sub> = number extra full length leaves N<sub>g</sub> = number of graduated leaves including master leaf b = width of each leaf t = thickness of each leaf l = half-length of leaf spring or length of cantilever W = total load transmitted to the axle of vehicle Also, W = 2F; F = F<sub>f</sub> + F<sub>g</sub>; N = N<sub>f</sub> + N<sub>g</sub>; First consider the half portion of the first group of the leaf spring which looks like a cantilever beam containing N<sub>g</sub> number of graduated leaves including master leaf. since the load is applied at the free end of the master leaf, in order to find out the induced stress and deflection, the leaves are assumed to be arranged in such way that they can be treated as a triangular plate. For this the individual leaves are separated and the master leaf is placed at the center. Then the second leaf is cut longitudinally into two halves, each of width (b/2) and placed on each side of master leaf. The same method is adopted for other also. the resultant is approximately a triangular plate of thickness (t) and a maximum width at the support as (N<sub>g</sub> \* b) the maximum stress produced at the fixed end is given by

$$S_{bg} = \frac{M}{Z} = \frac{MY}{I} = \frac{F_g \cdot l \cdot t/2}{(N_g b) t^3 \frac{1}{12}} = \frac{6F_g \cdot l}{N_g b t^2}$$

The maximum deflection produced at the free end of triangular plate is given by

$$\delta_g = \frac{F_g l^3}{2EI} = \frac{F_g \cdot l^3}{2E \left( \frac{N_g b t^3}{12} \right)} = \frac{6F_g l^3}{EN_g b t^3} = S_{bg} \frac{l^2}{Et}$$

Similarly, the second group of leaves i.e. the extra full length leaves can be treated as rectangular plate of thickness (t) and uniform width (N<sub>f</sub>b). The maximum induced bending stress at fixed end,

$$\sigma_f = \frac{MY}{I} = \frac{(F_f \cdot l) (t/2)}{\frac{1}{12} (N_f b) t^3} = \frac{6F_f l}{N_f b t^2}$$

The deflection at the free end (i.e. load point) is given by

$$\delta_f = \frac{F_f \cdot l^3}{3EI} = \frac{F_f \cdot l^3}{3E \left[ \frac{1}{12} (N_f b) t^3 \right]} = \frac{4F_f l^3}{EN_f b t^3} = \frac{2}{3} \sigma_{\max} \frac{l^2}{Et}$$

Since the leaf spring is the combined form of full length leaves and graduated leaves, the deflection produced in both types of leaves are same.

$$\text{i.e. } \delta_g = \delta_f \text{ (or) } \frac{2}{3} \sigma_f \frac{l^2}{Et} = \sigma_g \frac{l^2}{Et}$$

$$\sigma_f = \frac{3}{2} \sigma_g$$

From the above equation we can understand that when the leaf spring is deflected, the full length leaves are more stressed than graduated leaves by 50%. Due to this extra stress the life of full length leaves are reduced comparing to the life of graduated leaves which is undesirable and hence the stress induced in both types of springs are made equal by providing a special arrangement in the spring. Suppose the nipping is not adopted, then stresses induced in full length leaves and graduated leaves are different and they may be calculated as follow.

For such case,  $\sigma_f = \frac{3}{2} \sigma_g$

$$\text{i.e. } \frac{6F_f l}{N_f b t^2} = \frac{3}{2} \frac{6F_g l}{2N_g b t^2}$$

Cancelling common parameters, we get

$$\frac{F_f}{N_f} = \frac{3F_g}{2N_g}$$

$$\text{i.e. } \frac{F - F_g}{N_f} = \frac{3F_g}{2N_g}$$

$$F = \frac{3F_g \cdot N_f}{2N_g} + F_g = F_g \left[ \frac{3N_f + 2N_g}{2N_g} \right]$$

$$\text{Or } F_g = \left[ \frac{2N_g}{3N_f + 2N_g} \right] F$$

Now  $\sigma_g$  = Bending stress in graduated leaves

$$= \frac{6F_g l}{N_g b t^2} = \frac{6}{N_g b t^2} \left[ \frac{2N_g}{3N_f + 2N_g} \right] Fl = \frac{12 Fl}{bt^2 (3N_f + 2N_g)}$$

And  $F_f$  = bending stress induced in full length leaves,

$$\frac{3}{2} \sigma_g = \frac{3}{2} \left[ \frac{12 Fl}{bt^2 (3N_f + 2N_g)} \right] = \frac{18 Fl}{bt^2 (3N_f + 2N_g)}$$

Deflection, which is common to full length and graduated leaves is given by

$$\delta = \frac{2\sigma_g l^2}{3Et} \text{ or } \frac{\sigma_g l^2}{Et}$$

$$= \frac{2 * l^2 \left[ \frac{18 Fl}{bt^2(3N_f + 2N_g)} \right]}{3 Et}$$

$$= \frac{12 Fl^3}{Ebt^3(3N_f + 2N_g)}$$

We have known that there is a method to equalize the stress in all the leaves by providing a gap called nip, in between full length and graduated leaves, before joining tightly by the clipping bolts, and if the clip bolts are tightened, the full length and graduated leaves will bend in opposite directions and the initial stress due to this bending may be relieved or increased in order to equalize the stress in both types of leaves. The value of nip (C), clipping load and the common stress in all the leaves may be obtained from the following relations.

$$\text{We know that } \sigma_g = \frac{6F_g l}{N_g b t^2} \text{ and } \sigma_f = \frac{6F_f l}{N_f b t^2}$$

In this case since  $\sigma_g = \sigma_f$ , we may,

$$\frac{6F_g l}{N_g b t^2} = \frac{6F_f l}{N_f b t^2} \text{ or } \frac{F_f}{N_f} = \frac{F_g}{N_g},$$

and this is equal to  $\frac{F}{N}$ .

Where  $F = F_g + F_f$  and  $N = N_f + N_g$

$$\text{i.e. } \frac{F_f}{N_f} = \frac{F_g}{N_g} = \frac{F}{N}$$

Hence the stress induced in all leaves,

$$\sigma = \frac{6 Fl}{N b t^2}$$

When the leaf spring is maximum deflected, the nip,

$C = \delta_g - \delta_f$   
(graduated leaves will deflect more than the full length leaves by the value of C)

$$C = \frac{6 F_g l^3}{N_g E b t^3} - \frac{4 F_f l^3}{N_f E b t^3}$$

$$= \frac{6 F l^3}{N E b t^3} - \frac{4 F l^3}{N E b t^3} = \frac{2 F l^3}{N E b t^3}$$

The load to be applied on the clip bolts to close the gap is determined by the fact that the gap is equal to the initial deflection of the full length leaves and graduated leaves.

know that  $C = \delta_n + \delta_{g1}$

$$\text{i.e. } \frac{2 F l^3}{N E b t^3} = \frac{4 \left( \frac{F_b}{2} \right) l^3}{N_f E b t^3} + \frac{6 \left( \frac{F_b}{2} \right) l^3}{N_g E b t^3}$$

$$\frac{2F}{N} = \frac{2F_b}{N_f} + \frac{3F_b}{N_g}$$

$$\frac{2F}{N} = F_b \left[ \frac{2}{N_f} + \frac{3}{N_g} \right]$$

$$F_b = \frac{2 F N_f N_g}{N(2 N_g + 3 N_f)}$$

By equating this clipping load ( $F_b$ ) to the tensile resisting strength of bolt, i.e.  $(\pi/4) d_c^2 s_t$  where  $s_t$  is the allowable tensile stress for

the bolt material, the diameter of the bolt may evaluate. The relationship between the radius of curvature, camber and span is given by the equation as  $2RY - Y^2 = l_1^2$  Where R = radius of curvature Y = Camber  $L_1$  = half span of spring Since 'Y' is very small compared to 'R', the value of  $Y^2$  can be neglected and hence  $R = \frac{l_1^2}{2Y}$  and camber is taken equal to the maximum deflection of the spring. Let 'N' be the total no. of leaves including master leaf. The no. of leaves not having eyes is (N-1) Let  $2l_1$  = span length of spring. a = width of band which is not having spring action and hence known as ineffective length. Then  $2l$  = Effective length =  $2l_1 - a$  For a leaf spring having one full length leaf excluding the master leaf, the length of all leaves are calculated as follows

$$\text{Length of smallest leaf} = \frac{\text{Effectivelength}}{N-1} + \text{ineffective length}$$

$$\text{(i.e. first leaf)} = \frac{2l}{N-1} * 1 + a$$

$$\text{Length of next leaf} = \frac{2l}{N-1} * 2 + \text{Length of third leaf}$$

= Similarly

$$\text{Length of (N-1)th leaf} = \frac{2l}{N-1} * (N-1) + a$$

In general, the length of any leaf, starting from smallest leaf is obtained as  $\frac{2l}{N-1} * r + a$ , where 'r' varies from 1 to (N-1).

The length of n<sup>th</sup> leaf which is master leaf

$$= \frac{2l}{N-1} * (N-1) + a + \pi(d+t)2$$

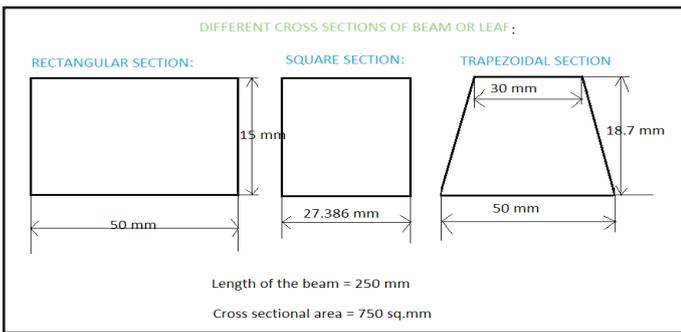
$$= 2l_1 + \pi(d+t)2$$

Where d = inside diameter of eye t = thickness of master leaf. Similarly, for a leaf spring having two full length leaves excluding the master leaf, the length of 'r<sup>th</sup>' leaf spring from smallest leaf is given by Length of 'r<sup>th</sup>' leaf =  $\frac{2l}{N-2} * r + a$  Where 'r' varies from 1 to (N-2). the (N-2)<sup>th</sup>, (N-1)<sup>th</sup> and N<sup>th</sup> leaves are having equal lengths, with an extra length for eyes of master leaf (N<sup>th</sup>).

### III. Designing Procedure For Leaf Spring

1. From the available data, note the loads to be supported, no. of springs required, space available and other working conditions.
2. Then choose a suitable material for the leaf spring and assume its design yield stress in bending. Usually the materials used for leaf springs are silicon steel and chromium manganese steel and the allowable stress is ranging from 400 to 600 n/m<sup>2</sup>.
3. Based on the requirement, adopt prestressing method if required. Usually this prestressing or nipping is preferable so as to make all the leaves to be equally stressed. Generally among all the leaves, adopt at least two leaves including master leaf as full length leaves and consider other leaves as graduated leaves.
4. Determine the stress induced in the corresponding leaves and also find out the deflection produced using proper equations. Similarly, evaluate the width and thickness of leaves. Find out the length of leaves, camber, clipping load, the radius of curvature etc. using proper relations. Draw a neat sketch of leaf spring.

## A. Dimensions of Taking Leaf Springs:



Force acting on spring is: 2000n

## IV. Theorital Calculations:

### A. Analysis of Mono Leaf Spring with Rectangular Crossscetion:

STEEL MATERIAL:REACTION FORCES AT NODE 1(N1):

$$F_x = 0F_y = 2000$$

Taking moments about node 1(N1):

$$M_z = 250 \times 2000M_z = 500000$$

Moment of inertia of a rectangular beam:

$$= bd^3/12 = (50 \times 15^3)/12 = 14062.5 \text{ mm}^4$$

Deflection of the cantilever beam with point load at the free end:

$$= (WL^3)/(3EI) = (2000 \times 250^3) / (3 \times 2 \times 10^5 \times 14062.5) = 3.7 \text{ mm}$$

Stress:  $= (6WL)/(bd^2)$

$$= (6 \times 2000 \times 250) / (50 \times 15^2)$$

$$= 266.66 \text{ N/mm}^2$$

### B. Square Cross Section

REACTION FORCES AT NODE 1(N1):

$$F_x = 0F_y = 2000$$

Taking moments about node 1(N1):

$$M_z = 250 \times 2000M_z = 500000$$

Moment of inertia of a SQUARE beam:

$$= bd^3/12 = (27.38 \times 27.38^3) / 12$$

$$= 46833.05 \text{ mm}^4$$

Deflection of the cantilever beam with point load at the free end:

$$= (WL^3)/(3EI)$$

$$= (2000 \times 250^3) / (3 \times 2 \times 10^5 \times 46833.05)$$

$$= 1.11 \text{ mm}$$

Stress:  $= (6WL)/(bd^2)$

$$= (6 \times 2000 \times 250) / (27.38 \times 27.38^2)$$

$$= 146.15 \text{ N/mm}^2$$

### C. Trapezium Cross Section

REACTION FORCES AT NODE 1(N1):

$$F_x = 0F_y = 2000$$

Taking moments about node 1(N1):

$$M_z = 250 \times 2000M_z = 500000$$

Moment of inertia of a TRAPEZIUM beam:

$$= h^3 (3a+b)/12$$

$$= 18.75^3 ((3 \times 30) + 50) / 12$$

$$= 76904.29 \text{ mm}^4$$

Deflection of the cantilever beam with point load at the free end:

$$= (WL^3)/(3EI)$$

$$= (2000 \times 250^3) / (3 \times 2 \times 10^5 \times 76904.29)$$

$$= 0.677 \text{ mm}$$

Stress:  $= (6WL)/h^2(3a+b)$

$$= (6 \times 2000 \times 250) / (18.75^2 ((3 \times 30) + 50))$$

$$= 60.95 \text{ N/mm}^2$$

## D. Analysis of Mono Leaf Spring With E-Glass Fiber Material:

### 2.7.2.1 RECTANGULAR CROSSSCETION:

REACTION FORCES AT NODE 1(N1):

$$F_x = 0F_y = 2000$$

Taking moments about node 1(N1):

$$M_z = 250 \times 2000M_z = 500000$$

Moment of inertia of a rectangular beam:

$$= bd^3/12 = (50 \times 15^3) / 12$$

$$= 14062.5 \text{ mm}^4$$

Deflection of the cantilever beam with point load at the free end:

$$= (WL^3)/(3EI) = (2000 \times 250^3) / (3 \times 80 \times 10^3 \times 14062.5)$$

$$= 9.25 \text{ mm}$$

Stress:  $= (6WL)/(bd^2)$

$$= (6 \times 2000 \times 250) / (50 \times 15^2)$$

$$= 266.66 \text{ N/mm}^2$$

### E. Square Cross Section

REACTION FORCES AT NODE 1(N1):

$$F_x = 0F_y = 2000$$

Taking moments about node 1(N1):

$$M_z = 250 \times 2000M_z = 500000$$

Moment of inertia of a SQUARE beam =  $bd^3/12$

$$= (27.38 \times 27.38^3) / 12$$

$$= 46833.05 \text{ mm}^4$$

Deflection of the cantilever beam with point load at the free end:

$$= (WL^3)/(3EI) = (2000 \times 250^3) / (3 \times 80 \times 10^3 \times 46833.05)$$

$$= 2.7 \text{ mm}$$

Stress =  $(6WL)/(bd^2) = (6 \times 2000 \times 250) / (27.38 \times 27.38^2)$

$$= 146.15 \text{ N/mm}^2$$

### F. Circular Cross Section:

REACTION FORCES AT NODE 1(N1):

$$F_x = 0F_y = 2000$$

Taking moments about node 1(N1):

$$M_z = 250 \times 2000M_z = 500000$$

Moment of inertia of a CIRCULAR beam:

$$= \pi d^4 / 64 = \pi \times 30.9^4 / 64 = 44762.3 \text{ mm}^4$$

Deflection of the cantilever beam with point load at the free end:

$$= (WL^3)/(3EI)$$

$$= (2000 \times 250^3) / (3 \times 80 \times 10^3 \times 44762.3)$$

$$= 2.9 \text{ mm}$$

Stress:  $= 32WL/\pi D^3$

$$= (32 \times 2000 \times 250) / (\pi \times 30.9^3) = 172.62 \text{ N/mm}^2$$

### G. Trapezium Cross Section:

REACTION FORCES AT NODE 1(N1):

$$F_x = 0F_y = 2000$$

Taking moments about node 1(N1):

$$M_z = 250 \times 2000M_z = 500000$$

Moment of inertia of a TRAPEZIUM beam:

$$= h^3 (3a+b)/12$$

$$= 18.75^3 ((3 \times 30) + 50) / 12$$

$$= 76904.29 \text{ mm}^4$$

Deflection of the cantilever beam with point load at the free end:

$$= (WL^3)/(3EI) = (2000 \times 250^3) / (3 \times 80 \times 10^3 \times 76904.29) = 1.69 \text{ mm}$$

Stress =  $(6WL)/h^2(3a+b)$

$$= (6 \times 2000 \times 250) / (18.75^2 ((3 \times 30) + 50))$$

$$= 60.95 \text{ N/mm}^2$$

## H. Analysis of mono leaf spring with carbon fiber material:

Rectangular cross section:

REACTION FORCES AT NODE 1(N1):

$$F_x = 0, F_y = 2000$$

Taking moments about node 1(N1):

$$M_z = 250 \times 2000, M_z = 500000$$

Moment of inertia of a rectangular beam:

$$= bd^3/12 = (50 \times 15^3)/12 = 14062.5 \text{ mm}^4$$

Deflection of the cantilever beam with point load at the free end:

$$\begin{aligned} &= (WL^3)/(3EI) \\ &= (2000 \times 250^3)/(3 \times 150 \times 10^3 \times 14062.5) \\ &= 4.9 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Stress} &= (6WL)/(bd^2) \\ &= (6 \times 2000 \times 250)/(50 \times 15^2) \\ &= 266.66 \text{ N/mm}^2 \end{aligned}$$

### 2.7.3.2 SQUARE CROSSSECTION:

REACTION FORCES AT NODE 1(N1):

$$F_x = 0, F_y = 2000$$

Taking moments about node 1(N1):

$$M_z = 250 \times 2000, M_z = 500000$$

Moment of inertia of a SQUARE beam =  $bd^3/12$

$$\begin{aligned} &= (27.38 \times 27.38^3)/12 \\ &= 46833.05 \text{ mm}^4 \end{aligned}$$

Deflection of the cantilever beam with point load at the free end:

$$\begin{aligned} &= (WL^3)/(3EI) \\ &= (2000 \times 250^3)/(3 \times 150 \times 10^3 \times 46833.0) \\ &= 1.48 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Stress} &= (6WL)/(bd^2) \\ &= (6 \times 2000 \times 250)/(27.38 \times 27.38^2) \\ &= 146.15 \text{ N/mm}^2 \end{aligned}$$

### I. Circular Crosssection

REACTION FORCES AT NODE 1(N1):

$$F_x = 0, F_y = 2000$$

Taking moments about node 1(N1):

$$M_z = 250 \times 2000, M_z = 500000$$

Moment of inertia of a TRAPEZIUM beam:

$$= \pi d^4/64 = \pi \times 30.9^4/64 = 44762.3 \text{ mm}^4$$

Deflection of the cantilever beam with point load at the free end:

$$\begin{aligned} &= (WL^3)/(3EI) \\ &= (2000 \times 250^3)/(3 \times 150 \times 10^3 \times 44762.3) \\ &= 1.55 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Stress} &= 32WL/\pi D^3 \\ &= (32 \times 2000 \times 250)/(\pi \times 30.9^3) \\ &= 172.62 \text{ N/mm}^2 \end{aligned}$$

### J. Trapezium Crosssection

REACTION FORCES AT NODE 1(N1):

$$F_x = 0, F_y = 2000$$

Taking moments about node 1(N1):

$$M_z = 250 \times 2000, M_z = 500000$$

Moment of inertia of a TRAPEZIUM beam:

$$\begin{aligned} &= h^3(3a+b)/12 = 18.75^3((3 \times 30) + 50)/12 \\ &= 76904.29 \text{ mm}^4 \end{aligned}$$

Deflection of the cantilever beam with point load at the free end:

$$\begin{aligned} &= (WL^3)/(3EI) \\ &= (2000 \times 250^3)/(3 \times 150 \times 10^3 \times 76904.29) \\ &= 0.09 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Stress} &= (6WL)/h^2(3a+b) \\ &= (6 \times 2000 \times 250)/18.75^2((3 \times 30) + 50) \\ &= 60.95 \text{ N/mm}^2 \end{aligned}$$

## V. Generic steps to solving any Problem in ANSYS

Like solving any problem analytically, you need to define (1) your solution domain, (2) the physical model, (3) boundary conditions and (4) the physical properties. You then solve the problem and present the results. In numerical methods, the main difference is an extra step called mesh generation. This is the step that divides the complex model into small elements that become solvable in an otherwise too complex situation. Below describes the processes in terminology slightly more attune to the software. Build Geometry: Construct a two or three dimensional representation of the object to be modeled and tested using the work plane coordinate system within ANSYS. Define Material Properties: Now that the part exists, define a library of the necessary materials that compose the object (or project) being modeled. This includes thermal and mechanical properties. Generate Mesh: At this point ANSYS understands the makeup of the part. Now define how the modeled system should be broken down into finite pieces. Apply Loads: Once the system is fully designed, the last task is to burden the system with constraints, such as physical loadings or boundary conditions. Obtain Solution: This is actually a step, because ANSYS needs to understand within what state (steady state, transient... etc.) the problem must be solved.

### Present the Results

After the solution has been obtained, there are many ways to present ANSYS' results, choose from many options such as tables, graphs, and contour plots.

## VI. Modal and Static Analysis

### A. Modal Analysis

Modal analysis is used to determine the vibration characteristics (natural frequencies and mode shapes) of a structure or a machine component while it is being designed. It can also serve as a starting point for another, more detailed, dynamic analysis, such as a transient dynamic analysis, a harmonic response analysis, or a spectrum analysis. Modal analysis is used to determine the natural frequencies and mode shapes of a structure. The natural frequencies and mode shapes are important parameters in the design of a structure for dynamic loading conditions. The general process for a modal analysis consists of these primary steps:

1. Build the model.
2. Apply loads and obtain the solution.
3. Expand the modes.
4. Review the results.

### B. Static Analysis on Leaf Springs:

#### A. For Rectangular Area of Cross Section

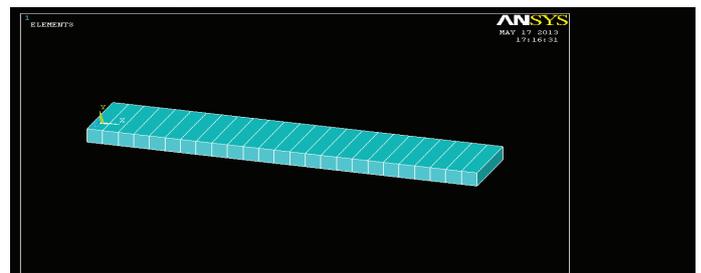


Fig. 6.1: Mono Leaf Spring of Rectangle Area of Cross Section

**B. For Steel Material**

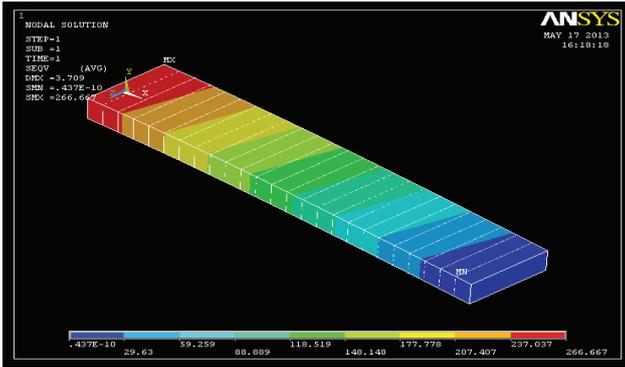


Fig. 6.2: Von-Mises Stress Plot of Steel MonoLeaf Spring

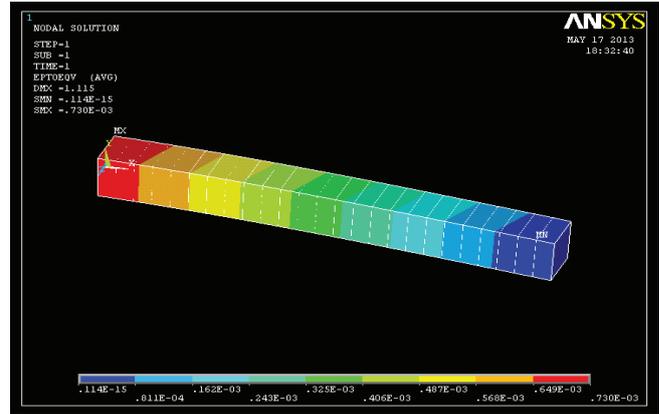


Fig. 6.6: Deflection Plot of Steel MonoLeaf Spring

**C. For E-Glass Fiber Material:**

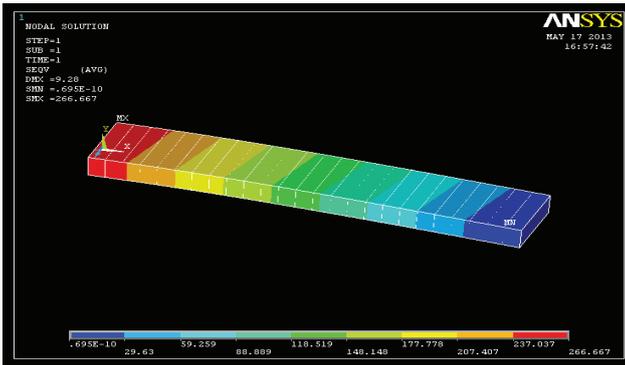


Fig. 6.3: Von-Mises Stress Plot of E-glass Fiber MonoLeaf Spring

**F. For E-Glass Fiber Material**

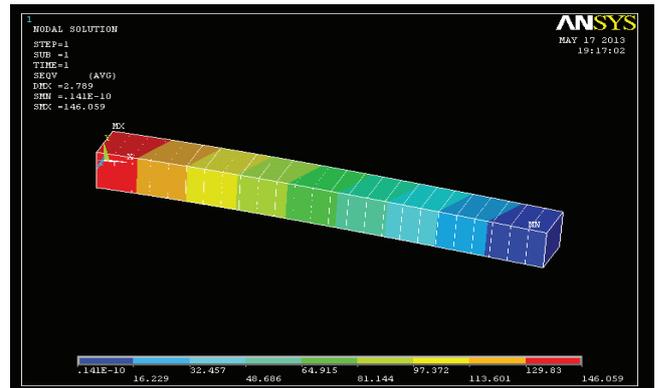


Fig. 6.7: Von-Mises Stress Plot of E-glass fiber Leaf Spring

**D. For Carbon Fiber Material**

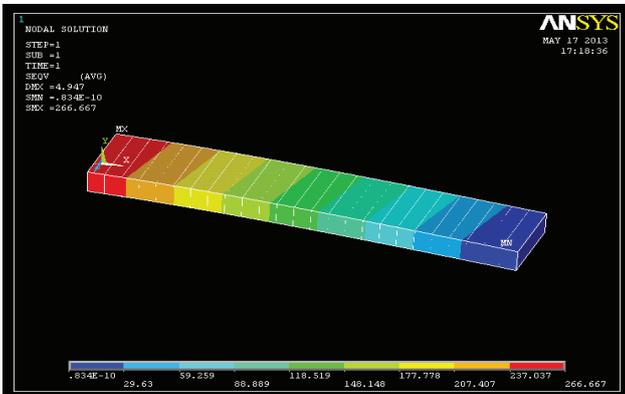


Fig. 6.4: Von-Mises Stress Plot of Carbon fiber MonoLeaf Spring

**G. For Carbon Fiber Material:**

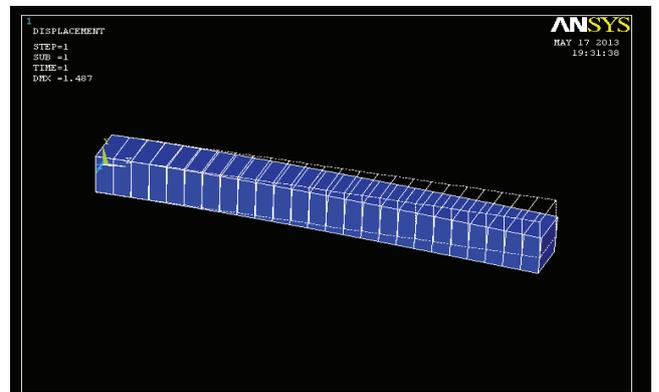


Fig. 6.8: Deformed and Undeformed Plot of Carbon fiber MonoLeaf Spring

**E. For Square area of cross section: For Steel Material:**

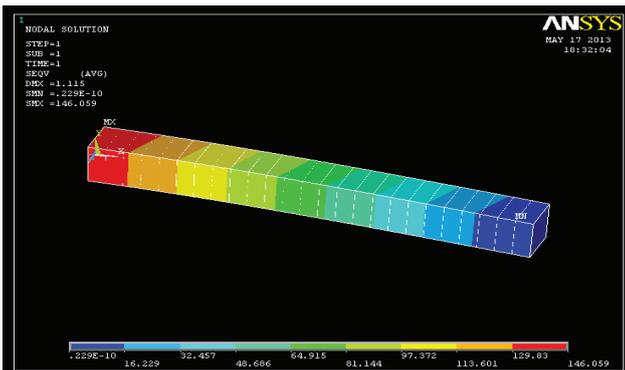


Fig. 6.5: Von-Mises Stress Plot of Steel MonoLeaf Spring

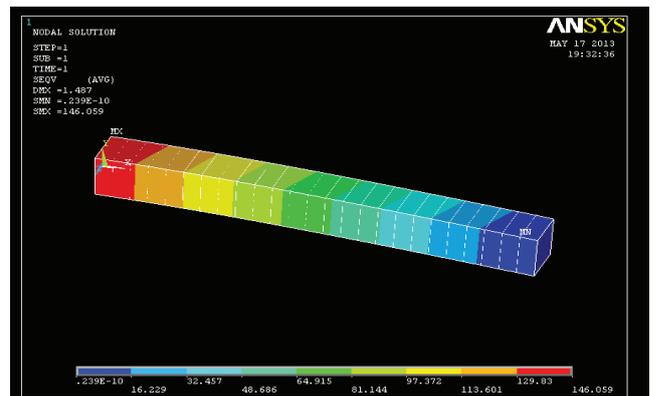


Fig. 6.9: Von-Mises Stress Plot of Carbon fiber MonoLeaf Spring

**H. For Circular Area of Cross Section**

For steel material:

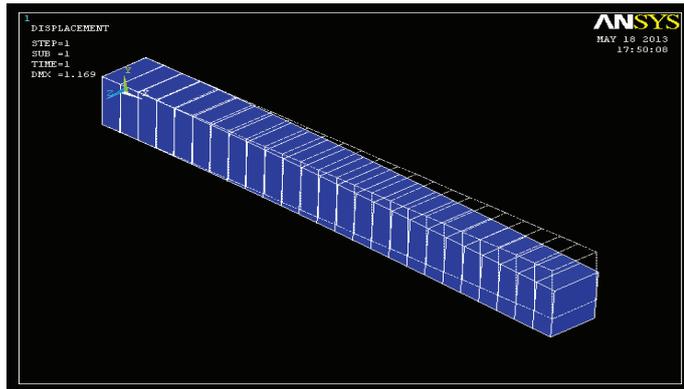


Fig. 6.10: Deformed and Undeformed Plot of Steel Mono Leaf Spring

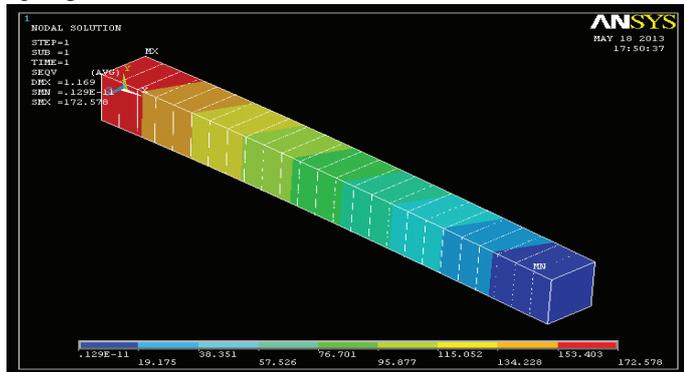


Fig. 6.11: Von-Mises Stress Plot of Steel Mono Leaf Spring

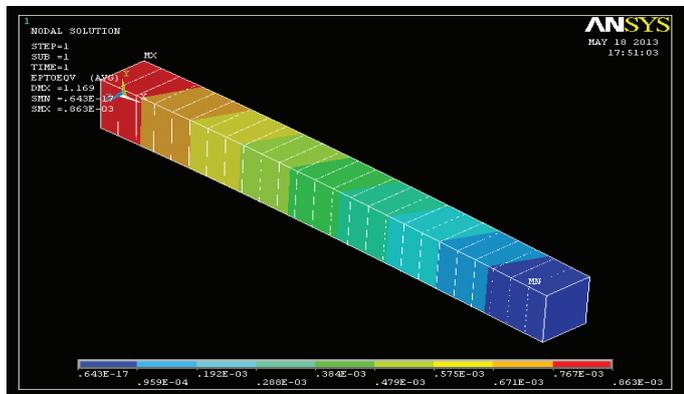


Fig.6.12: Deflection of Steel Mono Leaf Spring

**I. For E-Glass Fiber Material:**

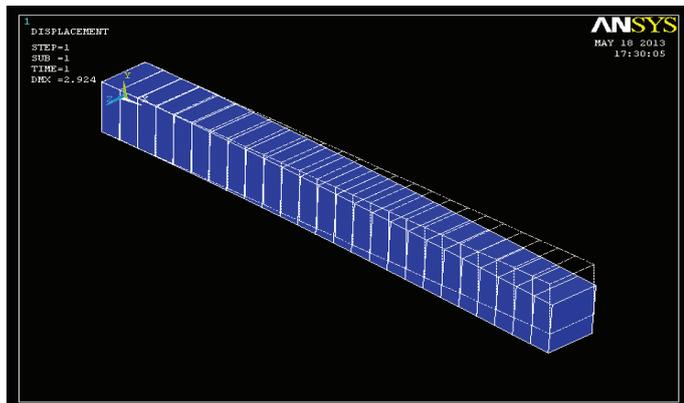


Fig. 6.13: Deformed and Undeformed Plot of E-glass fiber Mono Leaf Spring

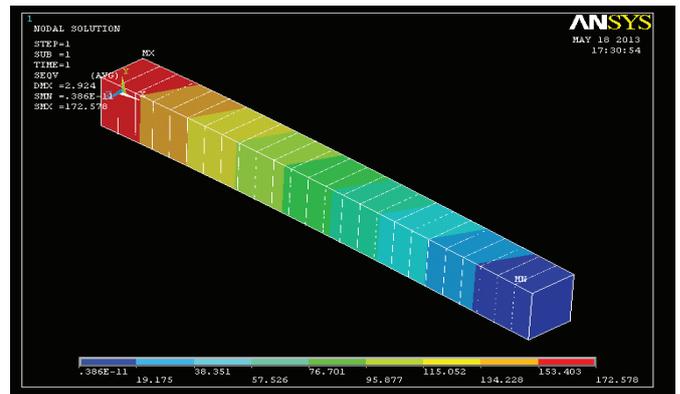


Fig. 6.14: Von-Mises Stress Plot of E-glass fiber Leaf Spring

**J. For Carbon Fiber Material**

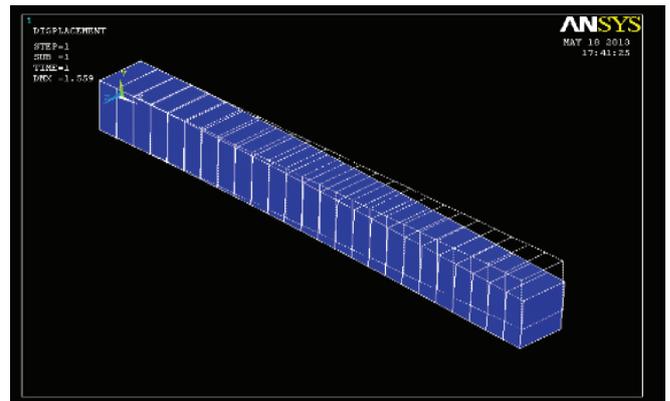


Fig. 6.15: Deformed and Undeformed Plot of Carbon fiber Mono Leaf Spring

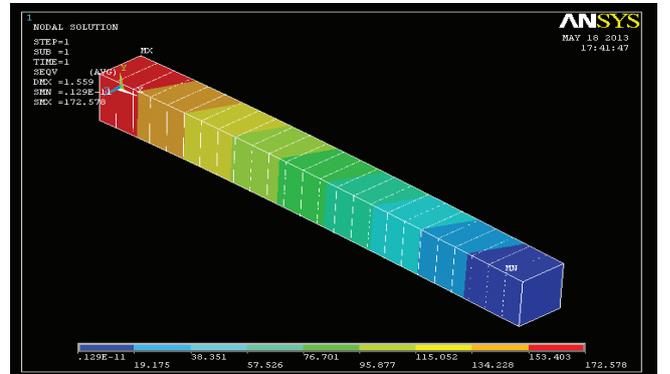


Fig. 6.16: Von-Mises Stress Plot of Carbon fiber Leaf Spring

**K. For Trepizoidal Area of Cross Ection:**

For Steel Material:

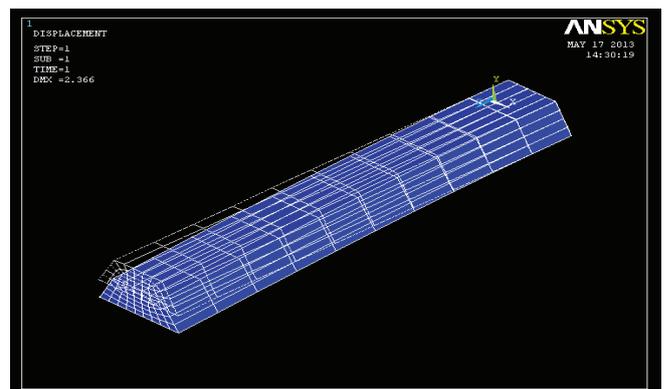


Fig. 6.17: Deformed and Undeformed Plot of Steel Mono Leaf Spring

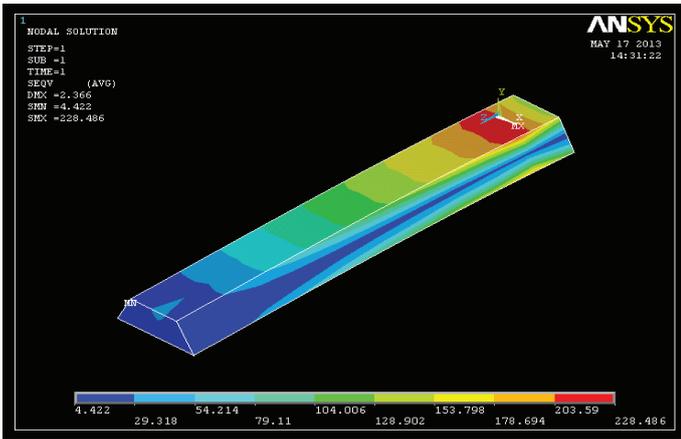


Fig. 6.18: Von-Mises Stress Plot of Steel mono Leaf Spring

**B. For Square Cross section:**

For Steel Material:

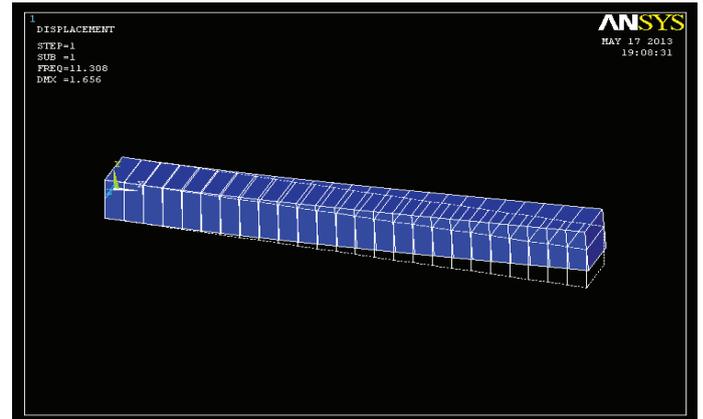


Fig 7.4: First Mode of Vibration

**VII. Modal Analysis on Leaf Springs**

**A. For Rectangular Cross Section**

**1. Steel Material**

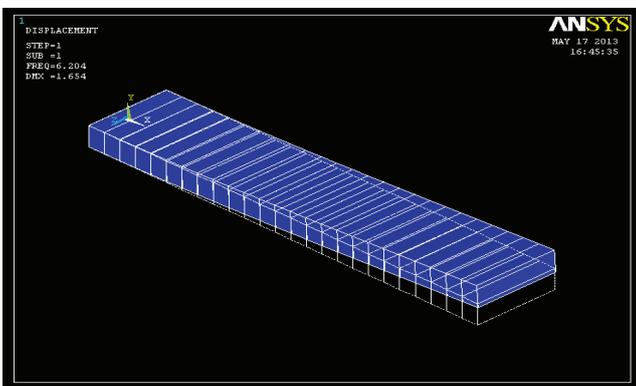


Fig 7.1: First Mode of Vibration

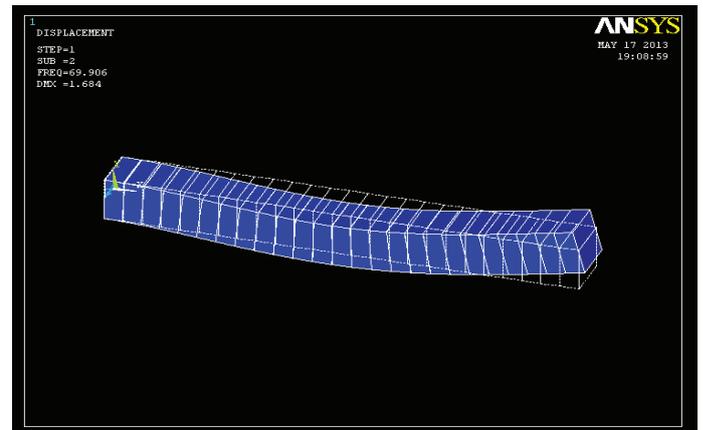


Fig 7.5: second Mode of Vibration

**C. For E-Glass fiber Material**

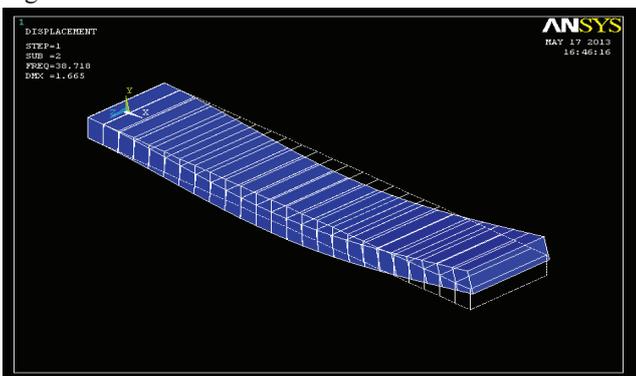


Fig 7.2: second Mode of Vibration

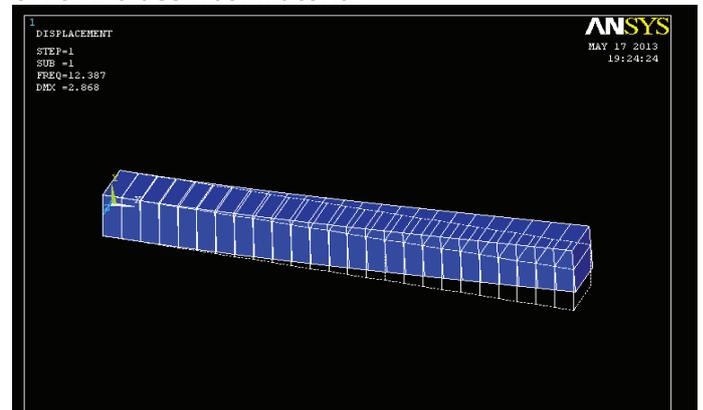


Fig 7.6 First Mode of Vibration

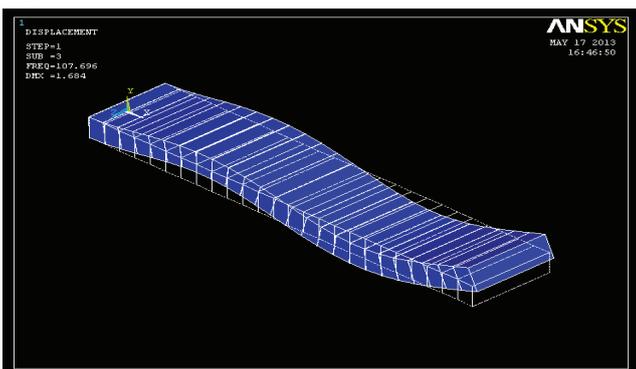


Fig 7.3: Third mode of vibration

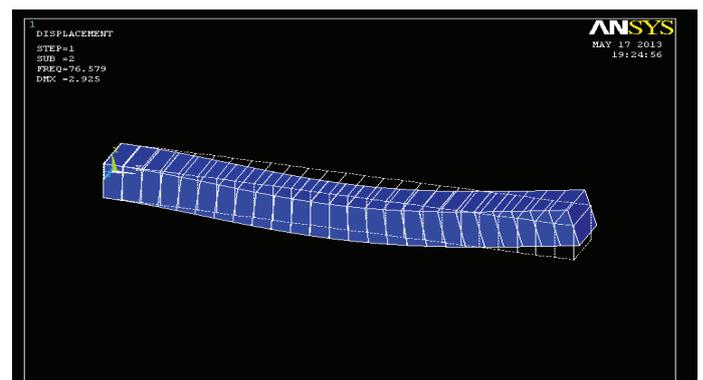


Fig 7.7: second Mode of Vibration

**D. For Carbon Fiber Material**

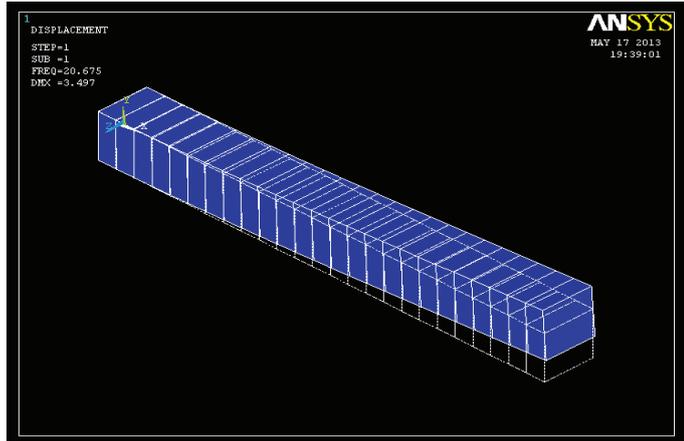


Fig 7.8: First Mode of Vibration

**VIII. Results and Conclusions**

**Results:** As per the manual calculations and analysis images  
**For Static Analysis:** For rectangular mono leaf spring

For steel		
Parameters	Theoretical results	ANSYS results
Deflection	3.7mm	3.7mm
Stress	266.66 N/mm <sup>2</sup>	266.66 N/mm <sup>2</sup>
For e-glass fiber		
Parameters	Theoretical results	ANSYS results
Deflection	9.25mm	9.25mm
Stress	266.66 N/mm <sup>2</sup>	266.66 N/mm <sup>2</sup>
For steel		
Parameters	Theoretical results	ANSYS results
Deflection	1.11mm	1.11mm
stress	146.15N/mm <sup>2</sup>	146.15 N/mm <sup>2</sup>
For carbon fiber		
Parameters	Theoretical results	ANSYS results
Deflection	1.48mm	1.48mm
stress	146.15 N/mm <sup>2</sup>	146.15 N/mm <sup>2</sup>

**For Nodal Analysis:** Rectangular cross section:forsteel e-glass fiber:

For steel		
Parameters	Theoretical results	ANSYS results
Deflection	0.677mm	0.677mm
stress	60.95 N/mm <sup>2</sup>	60.95 N/mm <sup>2</sup>
For e-glass fiber		
Parameters	Theoretical results	ANSYS results
Deflection	1.69mm	1.69mm
stress	60.95 N/mm <sup>2</sup>	60.95 N/mm <sup>2</sup>

For carbon fiber		
Parameters	Theoretical results	ANSYS results
Deflection	0.09mm	0.09mm
stress	60.95 N/mm <sup>2</sup>	60.95 N/mm <sup>2</sup>

**For Carbon Fiber:** Square Cross Section:for Steel:

FOR E-GLASS FIBER:

* Index of Data Sets on Results File				
Set	Time/Freq	Load Step	Substep	Cumulative
1	12.387	1	1	1
2	76.579	1	2	2
3	175.44	1	3	3

FOR CARBON FIBER:

CURCULAR CROSS SECTION:FOR STEEL:

** Index of Data Sets on Results File				
Set	Time/Freq	Load Step	Substep	cumulative
1	11.052	1	1	1
2	68.361	1	2	2
3	160.15	1	3	3

FOR E-GLASS FIBER:

* Index of Data Sets on Results File *				
Set	Time/Freq	Load Step	Substep	Cumulative
1	12.106	1	1	1
2	74.886	1	2	2
3	175.44	1	3	3

FOR CARBON FIBER:

* Index of Data Sets on Results File *				
Set	Time/Freq	Load Step	Substep	Cumulative
1	20.206	1	1	1
2	124.99	1	2	2
3	292.82	1	3	3

FOR TREPIZOIDAL CROSS SECTION:FOR STEEL:

** Index of Data Sets on Results File **				
Set	Time/Freq	Load Step	Substep	Cumulative
1	7.7972	1	1	1
2	16.983	1	2	2
3	48.675	1	3	3

**IX. Conclusion**

In our project we have analyzed a mono leaf spring using three different materials as steel, e-glass fiber ,carbon fiber and three different cross sections rectangular, square, trapezium. We have analyzed a mono leaf spring using ANSYS software. In ANSYS we had done static analysis on mono leaf spring to validate the stress on different materials having different cross sections. By comparing the stress and deflection values, the suitable material for mono leaf spring is carbon fiber having trapezium cross section. We have also done modal analysis on mono leaf spring to validate the frequency values on different materials having different cross sections. By observing the results steel material having less mode

values in different cross sections. So we conclude that by taking stress values in to consideration in design of mono leaf spring, carbon fiber having trapezium cross section gives good result. By taking mode values into consideration, the best material is steel for mono leaf spring.

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